

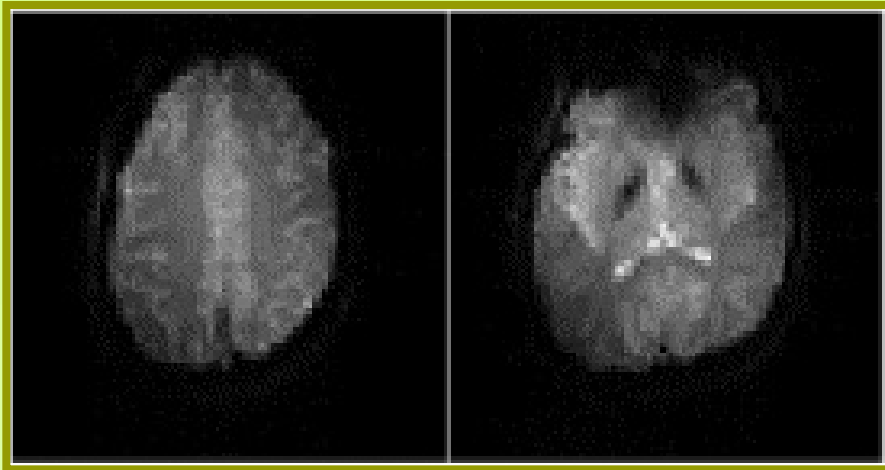
EPI distortion correction

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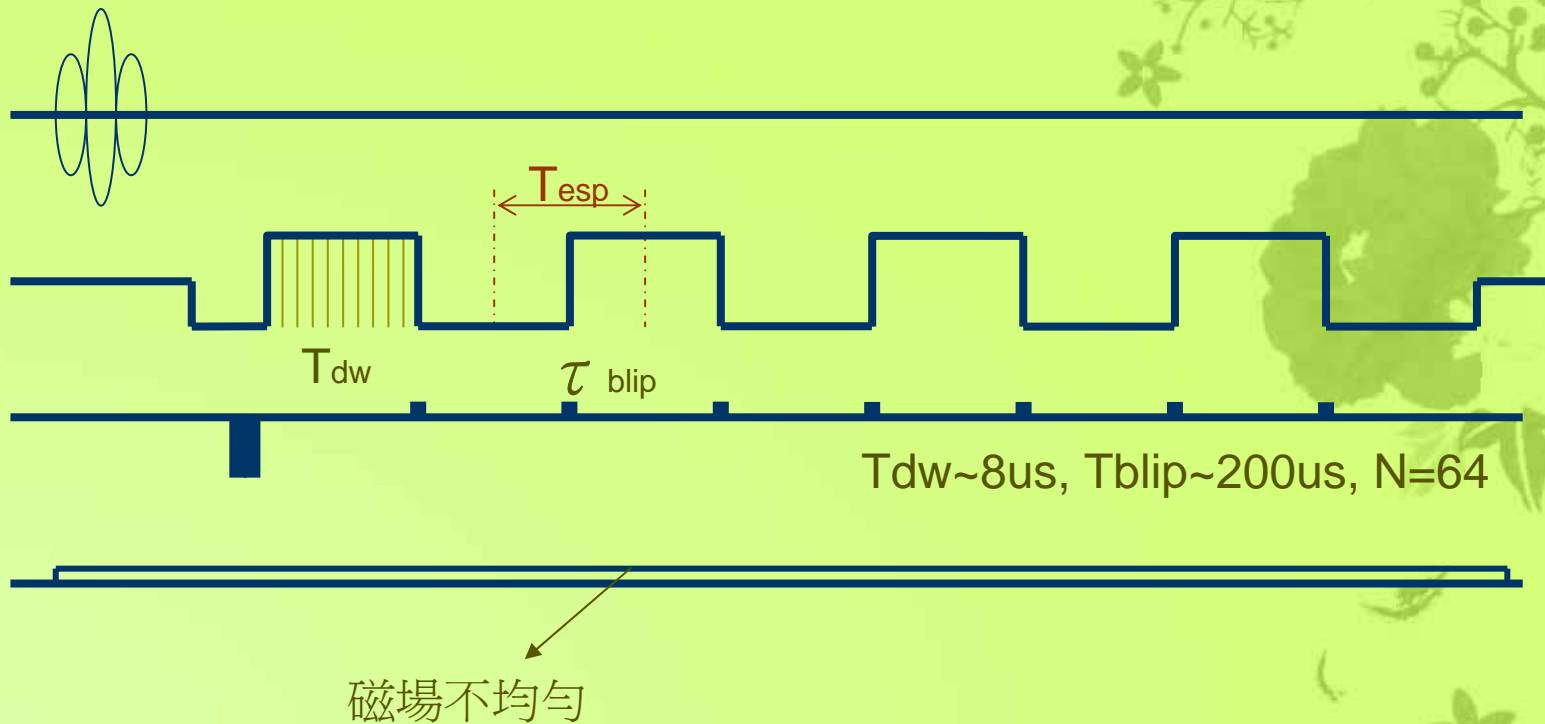
Distortion in EPI

- Susceptibility
 - Phase encoding direction
- Eddy current



Blipped EPI

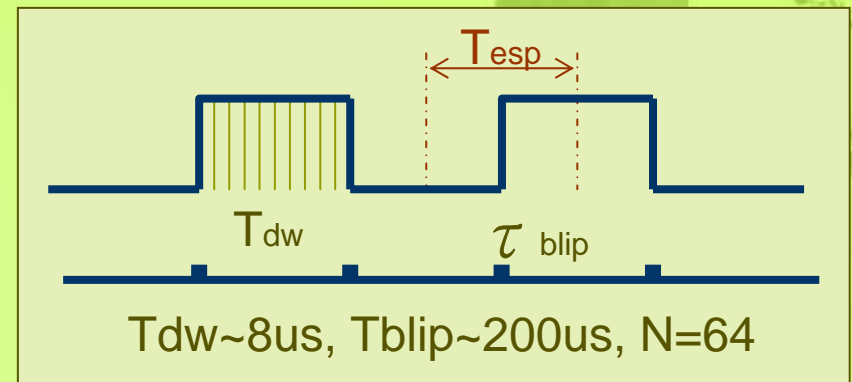
- Blipped EPI



Distortion in EPI

$$\begin{aligned}\phi_y(t) &= 2\pi\gamma[G_{blip}y\tau_{blip}N_y + \Delta B_0(\tau_{blip} + N_x T_{dw})N_y] \quad \text{y方向phase} \\ &= 2\pi\gamma\left[G_{blip} + \frac{\Delta B_0(\tau_{blip} + N_x T_{dw})}{y\tau_{blip}}\right]y\tau_{blip}N_y\end{aligned}$$

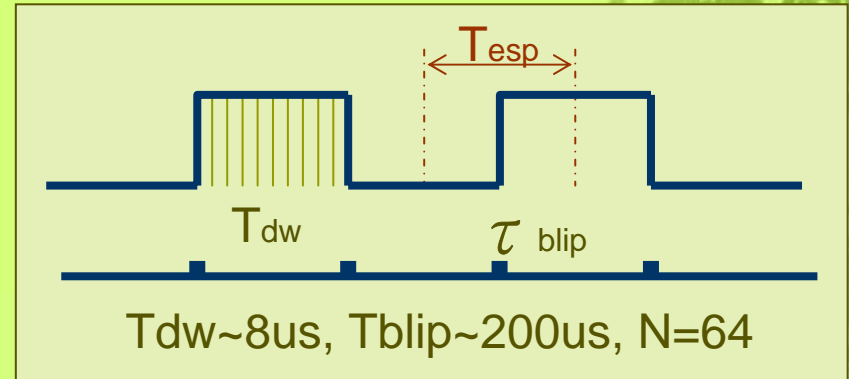
$$\Delta G_y(t) = \frac{\Delta B_0}{y} \frac{(\tau_{blip} + N_x T_{dw})}{\tau_{blip}}$$



Distortion in EPI

$$\Delta G_y(t) = \frac{\Delta B_0}{y} \frac{(\tau_{blip} + N_x T_{dw})}{\tau_{blip}}$$

$$BW_y = 1 / N_y (\tau_{blip} + N_x T_{dw})$$




$$\Delta y = \frac{\gamma \Delta G_y y}{BW_y} = \frac{\gamma \Delta B_0 (\tau_{blip} + N_x T_{dw})^2 N_y}{\tau_{blip}} \approx 3 \text{ pixel}$$

- N越大 distortion 越嚴重



Distortion Correction

- Field Map
 - Single-Shot Magnetic Field Mapping
 - K-Space Energy Spectrum Analysis
- 

- t=TE時的信號

$$S(TE) = \int_x \int_y \int_z \rho(x, y, z, 0) e^{-i2\pi\gamma \int_0^{TE} B_z(x, y, z, \tau) d\tau} dx dy dz$$

Phase term

$$\phi(TE) = 2\pi \gamma \int_0^{TE} B_z(x, y, z, \tau) d\tau$$

Multi-TE Correction

- 假設 B_z 的變化不大
 - 可忽略(time constant)

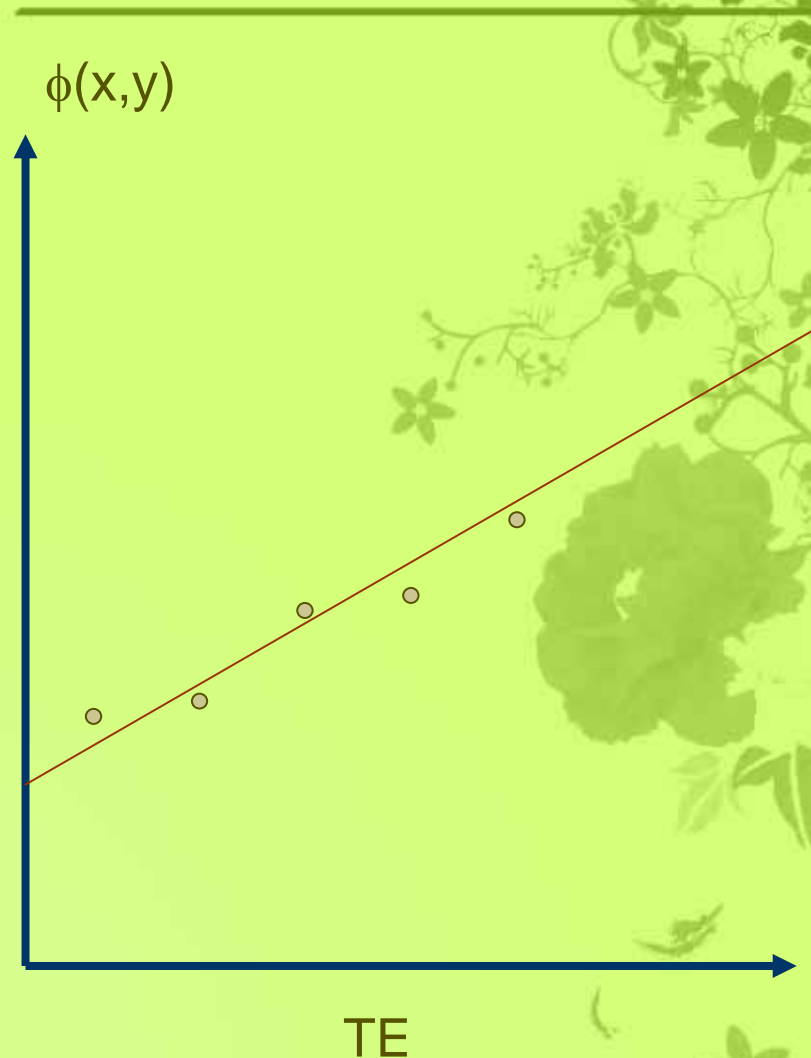
$$\begin{aligned}\Delta\phi &= \phi(TE + \Delta TE) - \phi(TE) \\ &= \gamma \int_0^{TE + \Delta TE} B_z(x, y, z, \tau) d\tau - \gamma \int_0^{TE} B_z(x, y, z, \tau) d\tau \\ &\approx \gamma B_z(x, y, z) \Delta TE\end{aligned}$$

$$B_z(x, y, z) = \frac{\Delta\phi}{\gamma\Delta TE}$$

Multi-TE Correction

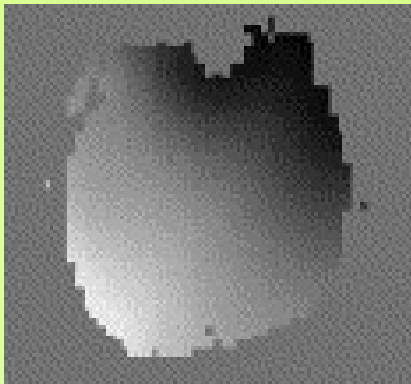
- 取多張(>2)EPI
- 影像相隔 ΔTE
- 每個pixel分別做 linear regression
- 計算field map

$$\Delta B_0(x, y) = \frac{\Delta \phi}{\gamma \Delta TE}$$



Multi-TE Correction

- Typical value :
 - 10 images & $\Delta TE \sim 0.8\text{ms}$
- Spatially smoothed
 - 2-D Gaussian kernel





Distortion Correction

- Field Map
 - Single-Shot Magnetic Field Mapping
 - K-Space Energy Spectrum Analysis
- 

Single-Shot multi-TE

- Single-Shot一次取完兩張不同TE的EPI
- 中間的k-space double sample

Single-Shot multi-TE

Single-Shot EPI

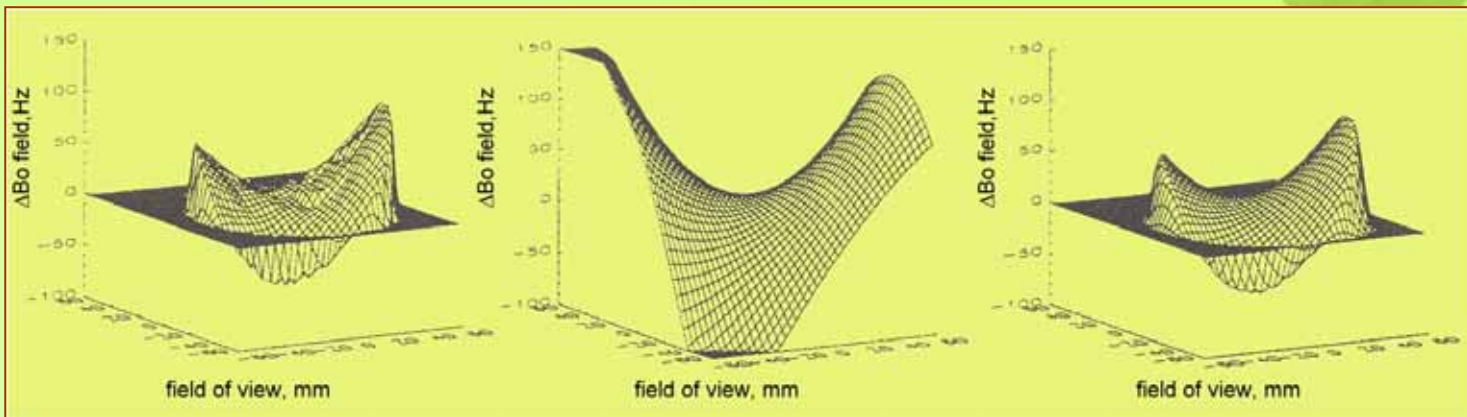
Field Map

- 中間sample兩次
- 經過原點兩次時間間隔 $\Delta TE \sim 1\text{ms}$
- 得到兩張不同TE的image
- 計算每個pixel的phase
- 計算field map

$$\Delta B_0(x, y) = \frac{\Delta\phi}{\gamma\Delta TE}$$

Field Map

- Mask
 - Threshold = 7% of max intensity
- Least-squares fitting
 - 7th order Polynomials



Correction

- 換算回k-space的位移

$$k'_y = k_y + \gamma \Delta B(x, y) T_{esp}$$

- 中間移兩倍距離
– (sample速度是兩倍)


$$\Delta y = N_y \Delta B_0(x, y) N_y T_{esp}$$

$$y = \tilde{y} - \Delta y$$

- 決定y
- 計算y' 的位置
- 把distortion y ' 的值填到正確的y

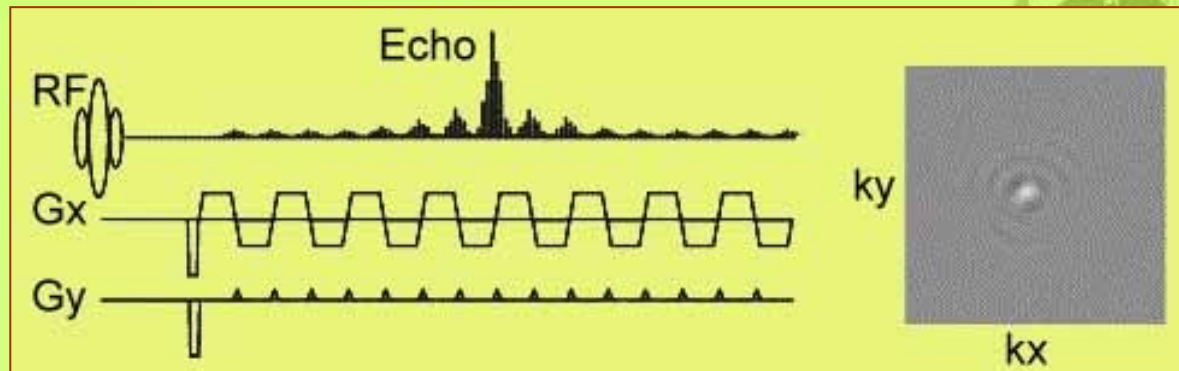


Distortion Correction

- Field Map
 - Single-Shot Magnetic Field Mapping
 - K-Space Energy Spectrum Analysis
- 

Echo Shifting

- 正常EPI

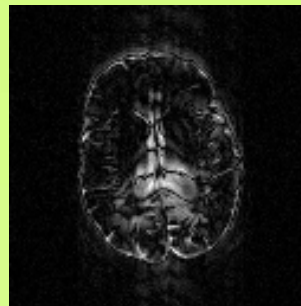
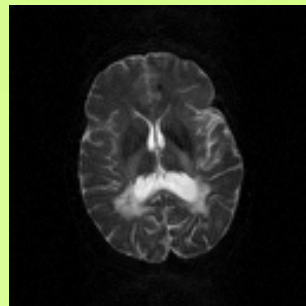
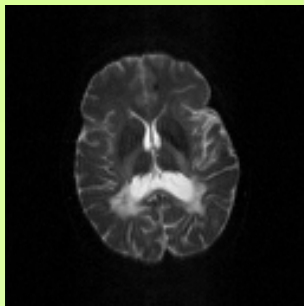
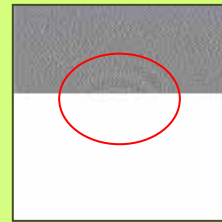
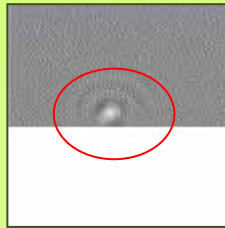


- EPI echo-shifting

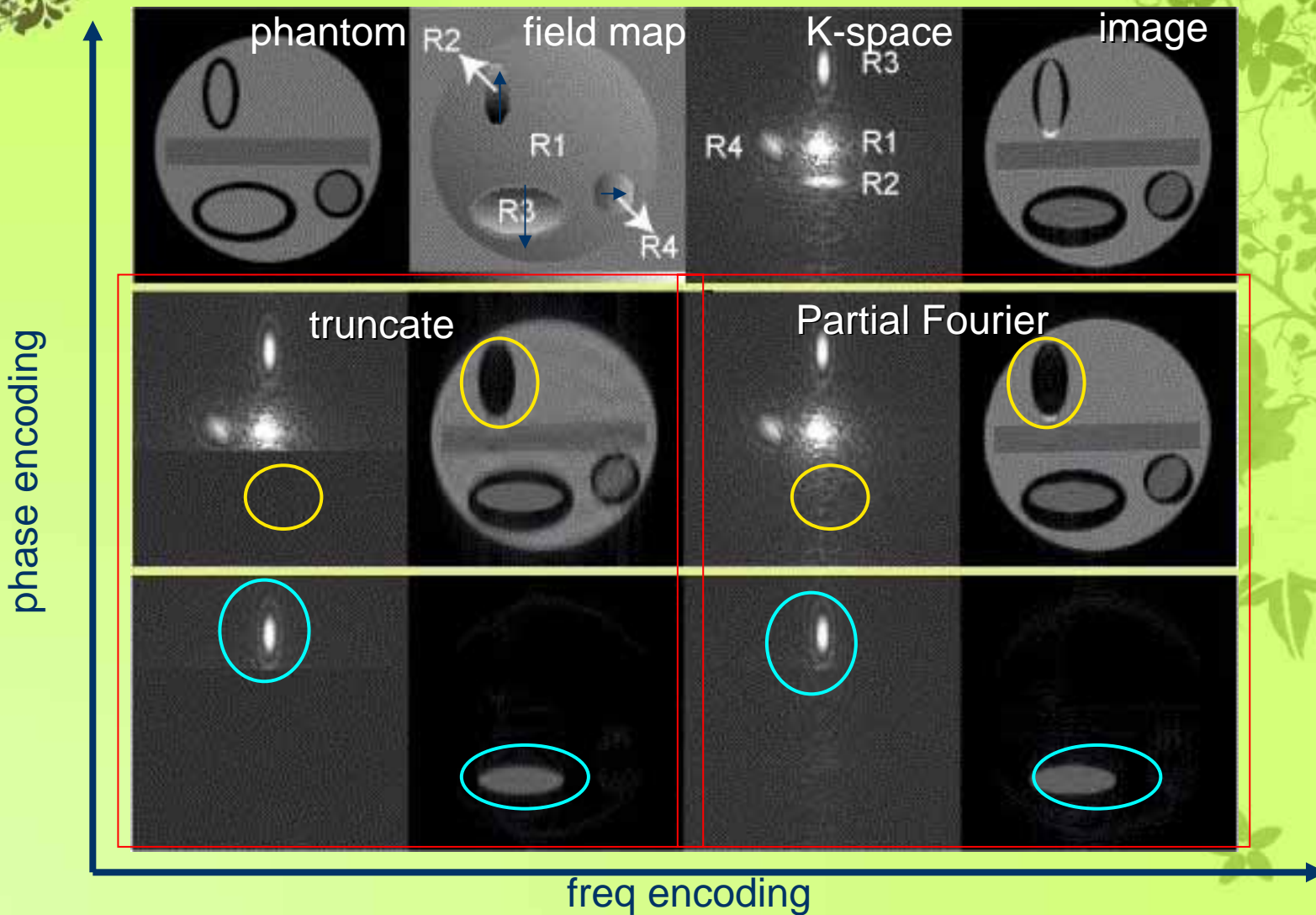


Partial k-space

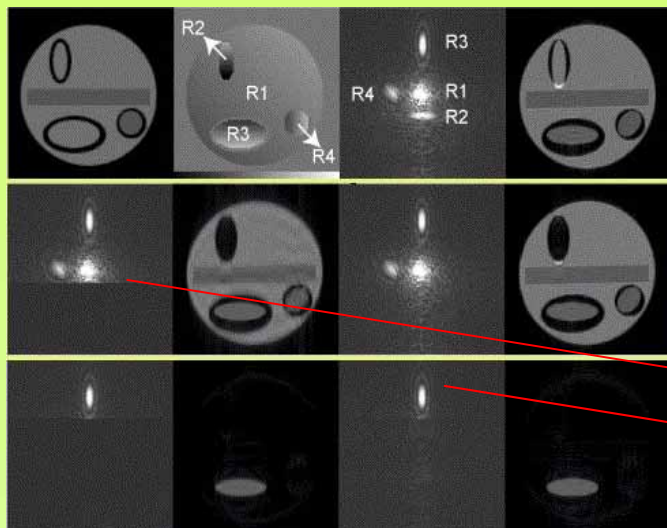
- Cuppen Method
- Truncated 超過 peak 無法復原



Truncate ky lines



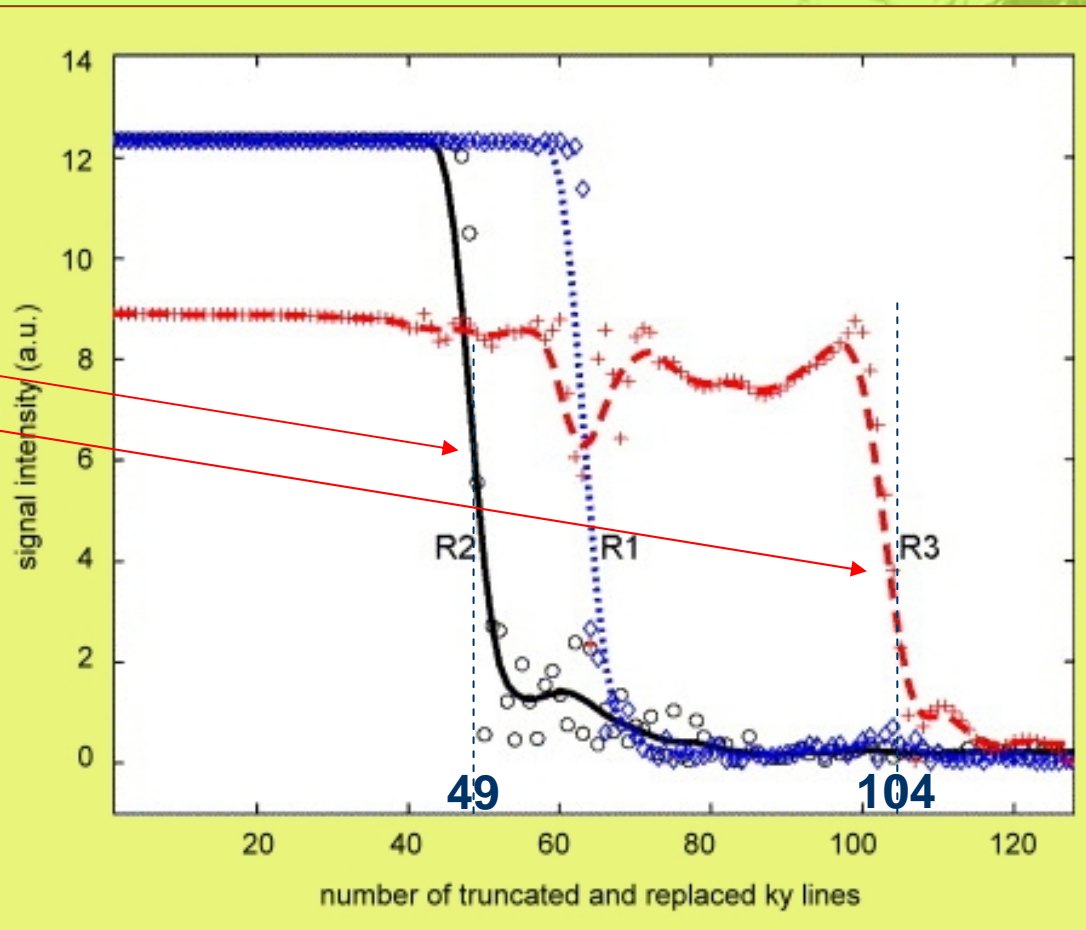
K-space energy spectrum



$$\Delta ky_{R1}=0$$

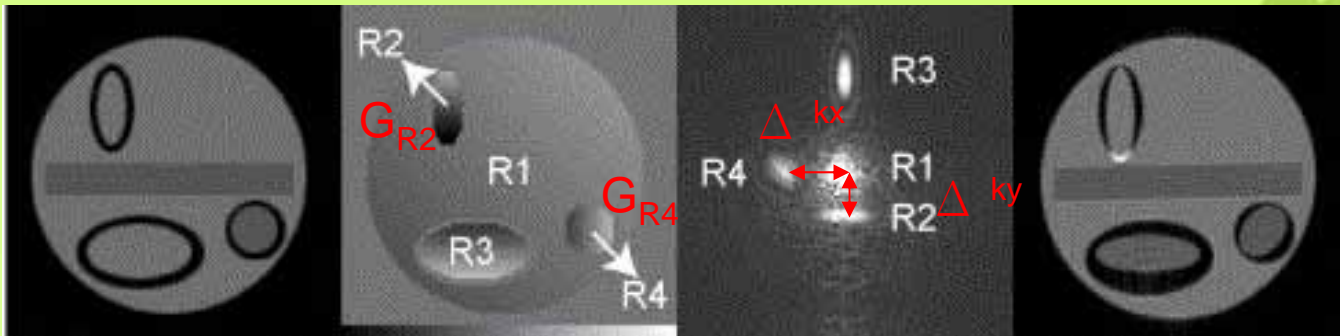
$$\Delta ky_{R2}=-16$$

$$\Delta ky_{R3}=39$$



k-space shift

$$\Delta_{R4}^{k_x}(0) = \frac{-G_{R4} TE_{des}}{G_x T_{dw} + G_{R4} T_{dw}} = \frac{-G_{R4} TE_{des}}{1/\gamma FOV_x + G_{R4} T_{dw}}$$



$$\Delta_{R2}^{k_y} = \frac{-G_{R2} TE_{des}}{G_{blip} \tau_{Blip} + G_{R2} T_{esp}} = \frac{-G_{R2} TE_{des}}{1/\gamma FOV_y + G_{R2} T_{esp}}$$

K-space shift

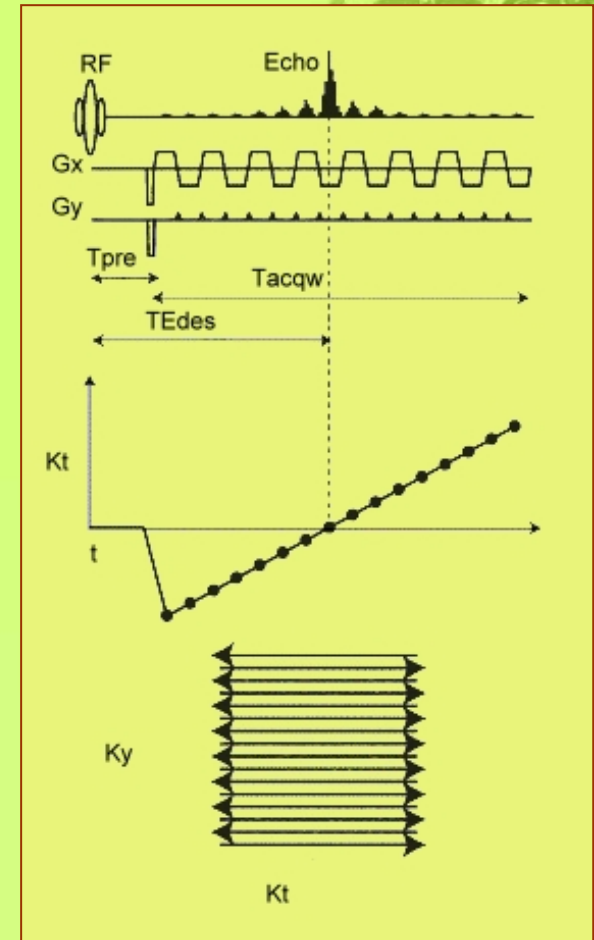
$$S(t) = \iint \rho(x, y) e^{i2\pi[\gamma \int_0^t G_x(t') x dt' + \gamma \int_0^t G_y(t') y dt']} e^{-t/T2^*} dx dy$$

$$= \iint \rho(x, y) e^{i2\pi[k_x(t)x + k_y(t)y]} e^{-t/T2^*} dx dy$$

$$k_x(t) = \int_0^t G_x(t') dt'$$

$$k_y(t) = \int_0^t G_y(t') dt'$$

$$k_y(TE_{des}) = \gamma \int_0^{TE_{des}} G_y(t) dt = 0$$

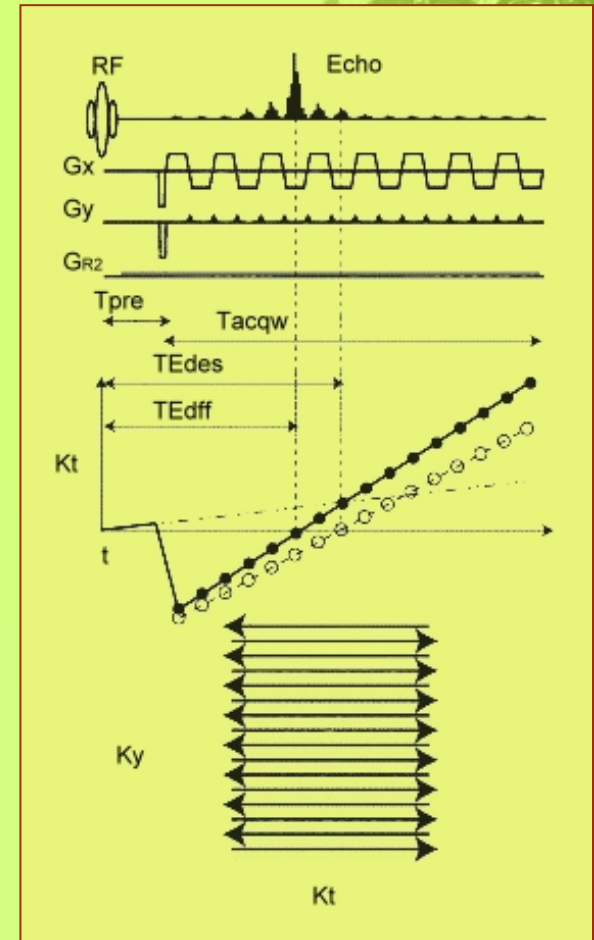
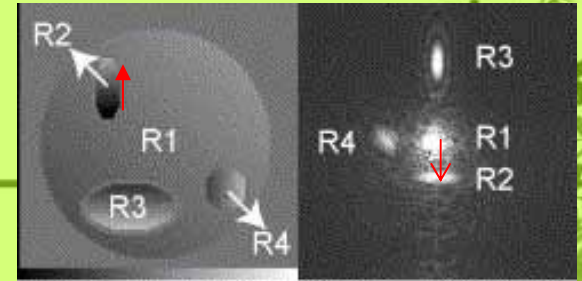


GR2 (1/3)

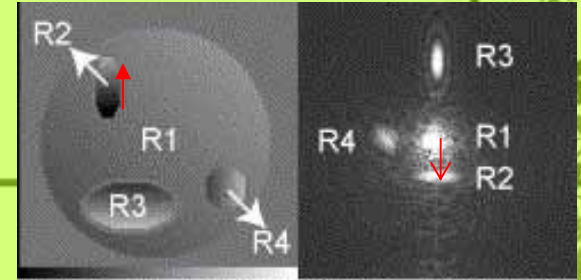
$$S_{R2}(t) = \iint_{R2} \rho(x, y) e^{i2\pi[k_x(t)x + k_y(t)y + \gamma G_{R2}t]} e^{-t/T2^*} dx dy$$

$$= \iint_{R2} \rho(x, y) e^{i2\pi[k_x(t)x + \tilde{k}_y(t, G_{R2})y]} e^{-t/T2^*} dx dy$$

$$\tilde{k}_y(t, G_{R2}) = k_y(t) + \gamma G_{R2}t \quad \text{distorted trajectory}$$



GR2 (2/3)



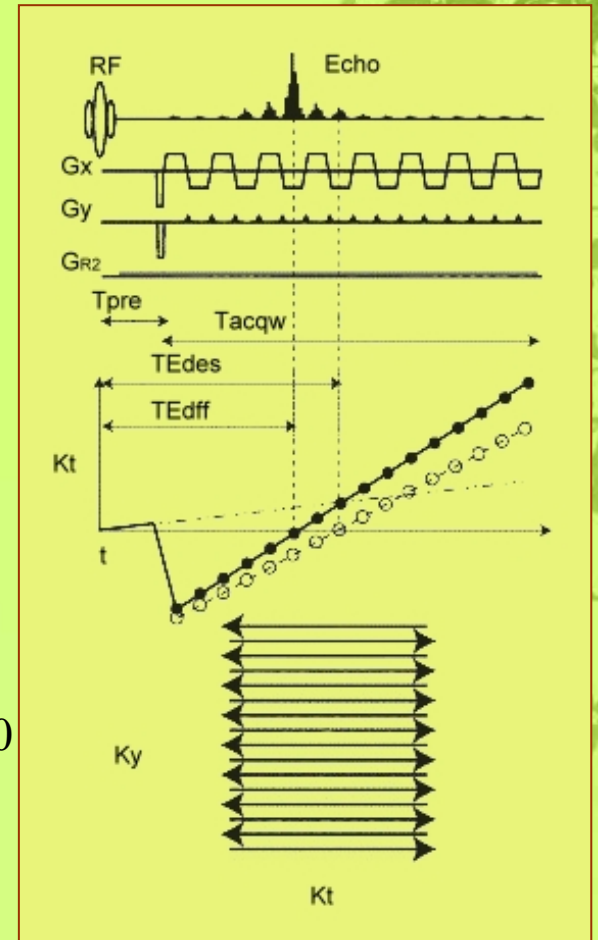
$\tilde{k}_y(t, G_{R2}) = k_y(t) + \gamma G_{R2} t$ distorted trajectory

$$\tilde{k}_y(TE_{\text{eff}}, G_{R2}) = 0 \quad k_y(TE_{\text{des}}) = 0$$

$$k_y(TE_{\text{des}}) + \int_{TE_{\text{des}}}^{TE_{\text{eff}}} G_y(t) dt + \gamma G_{R2} TE_{\text{eff}} = 0$$

$$\begin{aligned} \tilde{k}_y(TE_{\text{eff}}, G_2) &= k_y(TE_{\text{eff}}) + \gamma G_{R2} TE_{\text{eff}} \\ &= \int_0^{TE_{\text{eff}}} G_y(t) dt + \gamma G_{R2} TE_{\text{eff}} \\ &= \int_0^{TE_{\text{des}}} G_y(t) dt + \int_{TE_{\text{des}}}^{TE_{\text{eff}}} G_y(t) dt + \gamma G_{R2} TE_{\text{eff}} \\ &= k_y(TE_{\text{des}}) + \int_{TE_{\text{des}}}^{TE_{\text{eff}}} G_y(t) dt + \gamma G_{R2} TE_{\text{eff}} = 0 \end{aligned}$$

$$\int_{TE_{\text{des}}}^{TE_{\text{eff}}} G_y(t) dt = -G_{R2} TE_{\text{eff}}$$



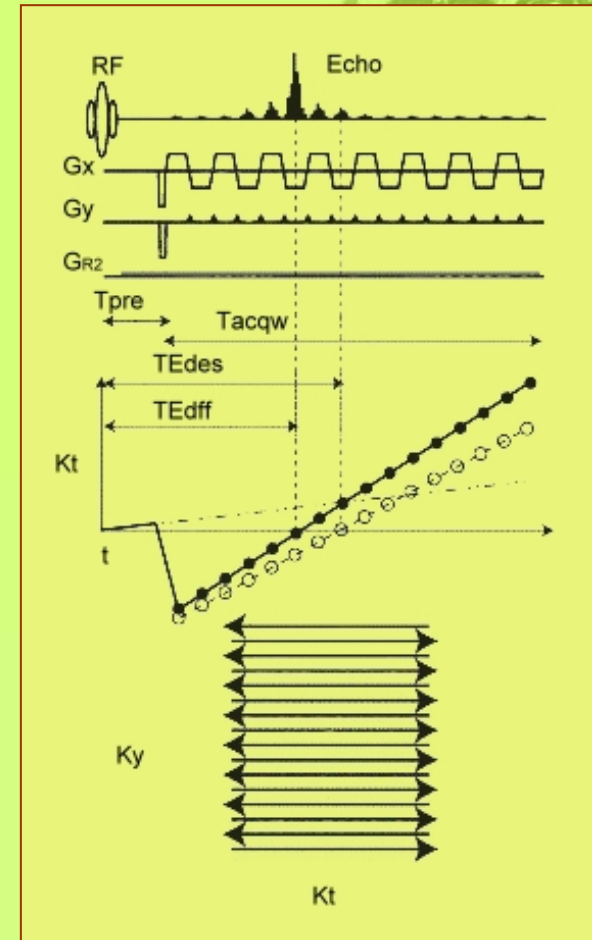
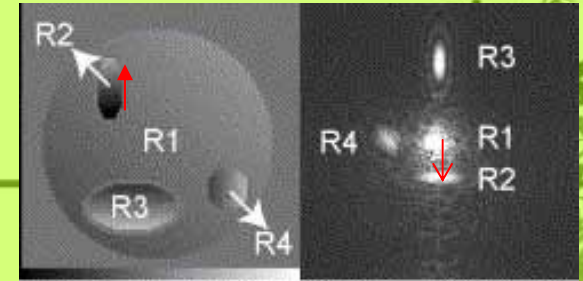
GR2 (3/3)

$$\int_{TE_{des}}^{TE_{eff}} G_y(t) dt = -G_{R2} TE_{eff}$$

$$G_{blip} \tau_{Blip} \Delta_{R2}^{k_y} = -G_{R2} (TE_{des} + \Delta_{R2}^{k_y} T_{esp})$$

$$\Delta_{R2}^{k_y} = \frac{-G_{R2} TE_{des}}{G_{blip} \tau_{Blip} + G_{R2} T_{esp}} = \frac{-G_{R2} TE_{des}}{\frac{1}{\gamma FOV_y} + G_{R2} T_{esp}}$$

$$\Delta_{R2}^{k_y} > 0 \quad (TE_{eff} > TE_{des}) \quad T_{esp} = \text{echo-spacing time}$$

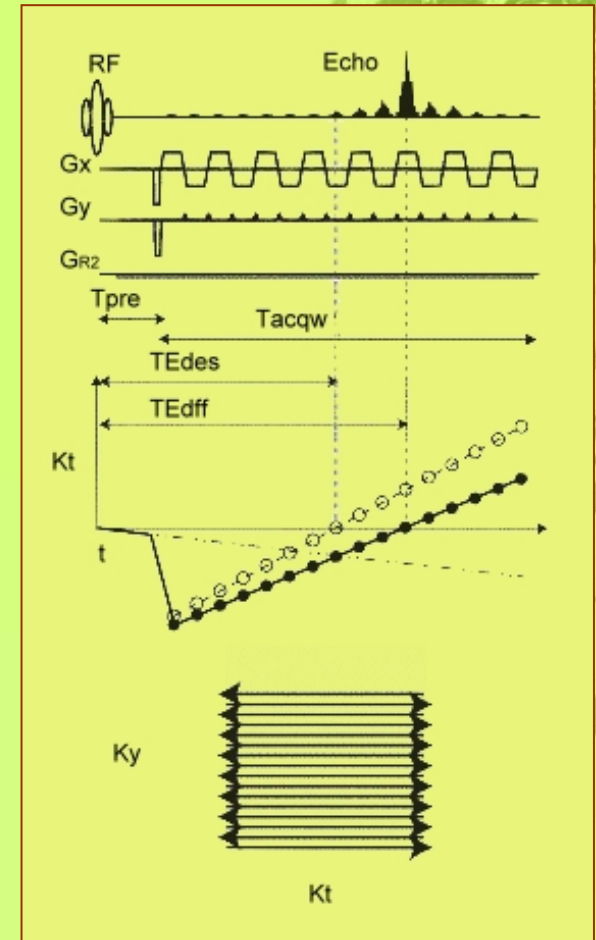
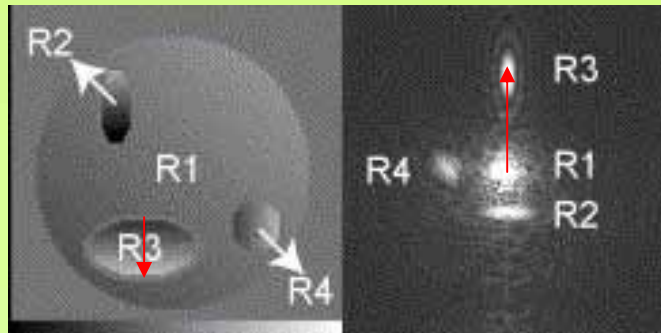


GR3

$$\Delta_{R3}^{k_y} = \frac{-G_{R3} TE_{des}}{G_{blip} \tau_{Blip} + G_{R3} T_{esp}} = \frac{-G_{R3} TE_{des}}{\frac{1}{\gamma FOV_y} + G_{R3} T_{esp}}$$

$$\Delta_{R3}^{k_y} < 0 \quad (TE_{eff} < TE_{des})$$

$$G_{R3} > 0$$



GR4 (1/2)

$$S_{R4}(t) = \iint_{R4} \rho(x, y) e^{i2\pi[k_x(t)x + k_y(t)y + \gamma G_{R4}t]} e^{-t/T2^*} dx dy$$

$$= \iint_{R4} \rho(x, y) e^{i2\pi[\tilde{k}_x(t, G_{R4}) + k_y(t)x]} e^{-t/T2^*} dx dy$$

$$\tilde{k}_x(t, G_{R4}) = k_x(t) + \gamma G_{R4}t$$

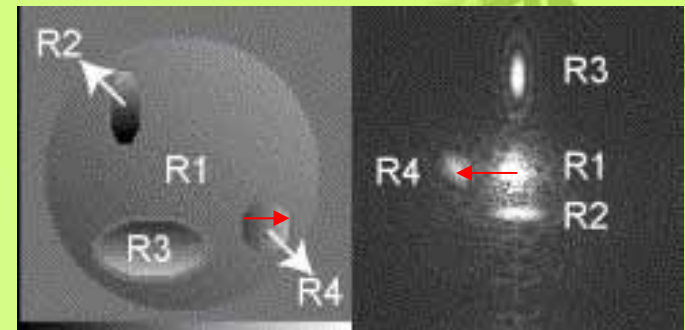
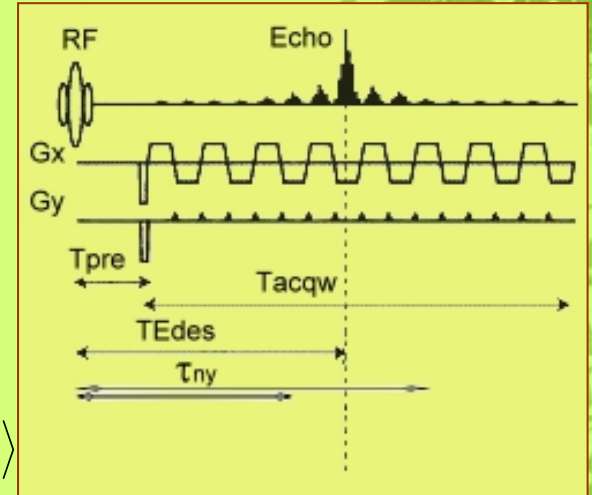
$$\tau_{n_y} = TE_{des} + n_y T_{esp}$$

$$\langle N_y/2 + 1 \leq n_y \leq N_y/2 \rangle$$

$$\tilde{k}_x(\tilde{\tau}_{n_y}) = 0 \quad k_x(\tau_{n_y}) = 0$$

$$k_x(\tau_{n_y}) + \gamma \int_{\tau_{n_y}}^{\tilde{\tau}_{n_y}} G_x(t) dt + \gamma G_{R4} \tilde{\tau}_{n_y} = 0$$

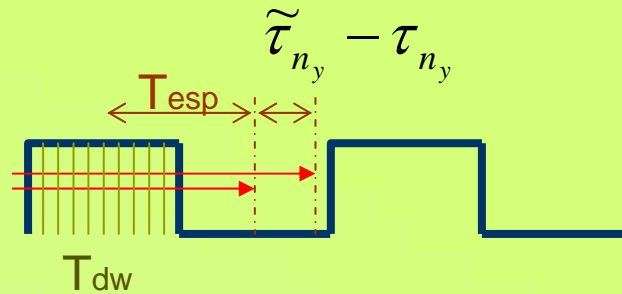
$$\gamma \int_{\tau_{n_y}}^{\tilde{\tau}_{n_y}} G_x(t) dt = -\gamma G_{R4} \tilde{\tau}_{n_y}$$



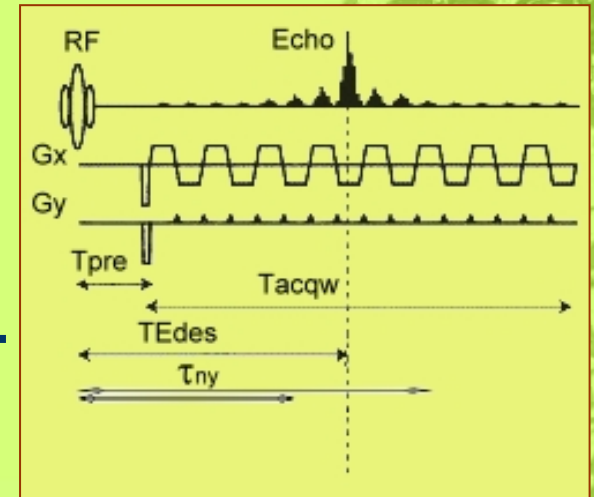
GR4 (2/2)

$$\int_{\tau_{n_y}}^{\tilde{\tau}_{n_y}} G_x(t) dt = -\gamma G_{R4} \tilde{\tau}_{n_y}$$

$$\Delta_{R4}^{k_x}(n_y) = \frac{\tilde{\tau}_{n_y} - \tau_{n_y}}{T_{dw}}$$

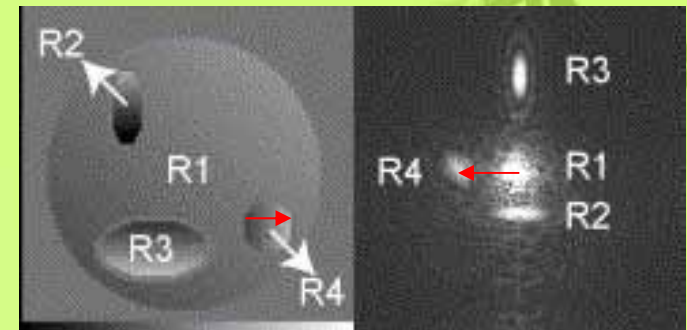


$$G_x T_{dw} \Delta_{R4}^{k_x}(n_y) = -G_{R4} (TE_{des} + n_y T_{esp} + \Delta_{R4}^{k_x}(n_y) T_{dw})$$



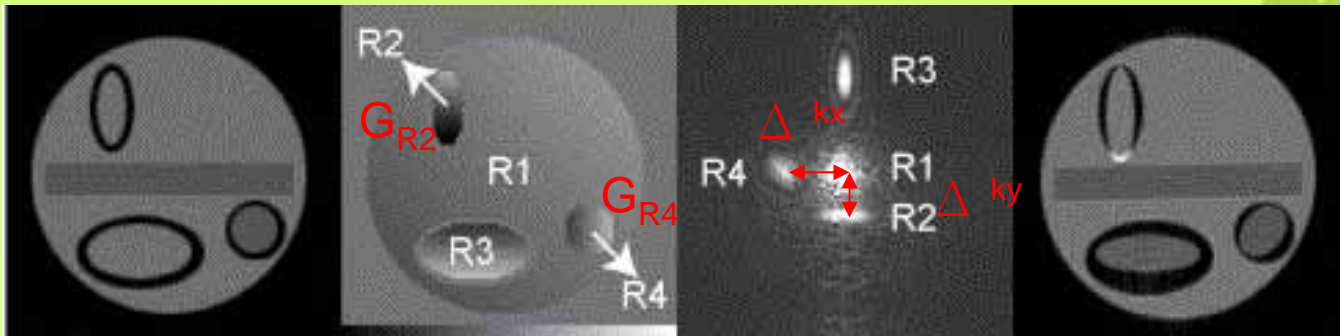
$$\Delta_{R4}^{k_x}(0) = \frac{-G_{R4} TE_{des}}{G_x T_{dw} + G_{R4} T_{dw}} = \frac{-G_{R4} TE_{des}}{\frac{1}{\gamma FOV_x} + G_{R4} T_{dw}}$$

$$\Delta_{R4}^{k_x}(0) = \frac{-G_{R4} (TE_{des} + \Delta_{R4}^{k_y} T_{esp})}{\frac{1}{\gamma FOV_x} + G_{R4} T_{dw}}$$



k-space shift

$$\Delta_{R4}^{k_x}(0) = \frac{-G_{R4} TE_{des}}{G_x T_{dw} + G_{R4} T_{dw}} = \frac{-G_{R4} TE_{des}}{1/\gamma FOV_x + G_{R4} T_{dw}}$$

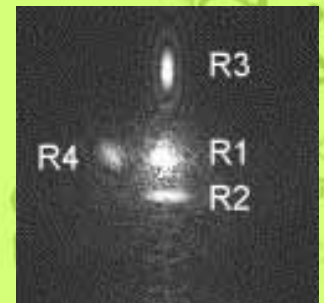
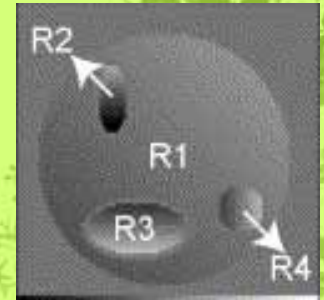


$$\Delta_{R2}^{k_y} = \frac{-G_{R2} TE_{des}}{G_{blip} \tau_{Blip} + G_{R2} T_{esp}} = \frac{-G_{R2} TE_{des}}{1/\gamma FOV_y + G_{R2} T_{esp}}$$

$$S(t) = \sum_{i=1}^N S_{Ri}(t)$$

$$= \sum_{i=1}^N \iint_{Ri} \rho(x, y) e^{i2\pi[k_x(t)x + k_y(t)y + \gamma G_{Ri}(x, y)t]} e^{-t/T2^*} dx dy$$

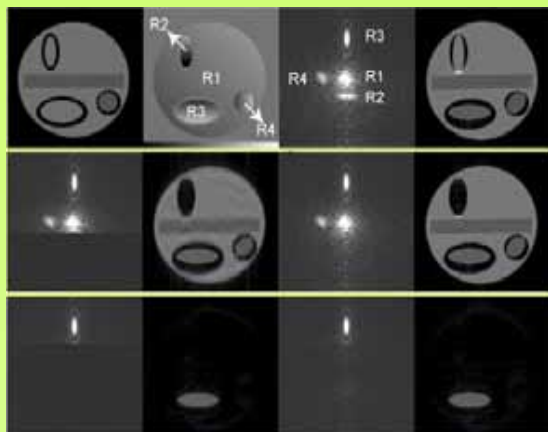
$$= \sum_{i=1}^N \iint_{Ri} \rho(x, y) e^{i2\pi[\tilde{k}_x(t, G_{Ri,x})x + \tilde{k}_y(t, G_{Ri,y})y]} e^{-t/T2^*} dx dy$$



$$\Delta_{Ri}^{k_y} = \frac{-G_{Ri,y} TE_{des}}{1/\gamma FOV_y + G_{Ri,y} T_{esp}}$$

$$\Delta_{Ri}^{k_x}(0) = \frac{-G_{Ri,x} (TE_{des} + \Delta_{Ri}^{k_y} T_{esp})}{1/\gamma FOV_x + G_{Ri,x} T_{dw}}$$

K-space energy spectrum

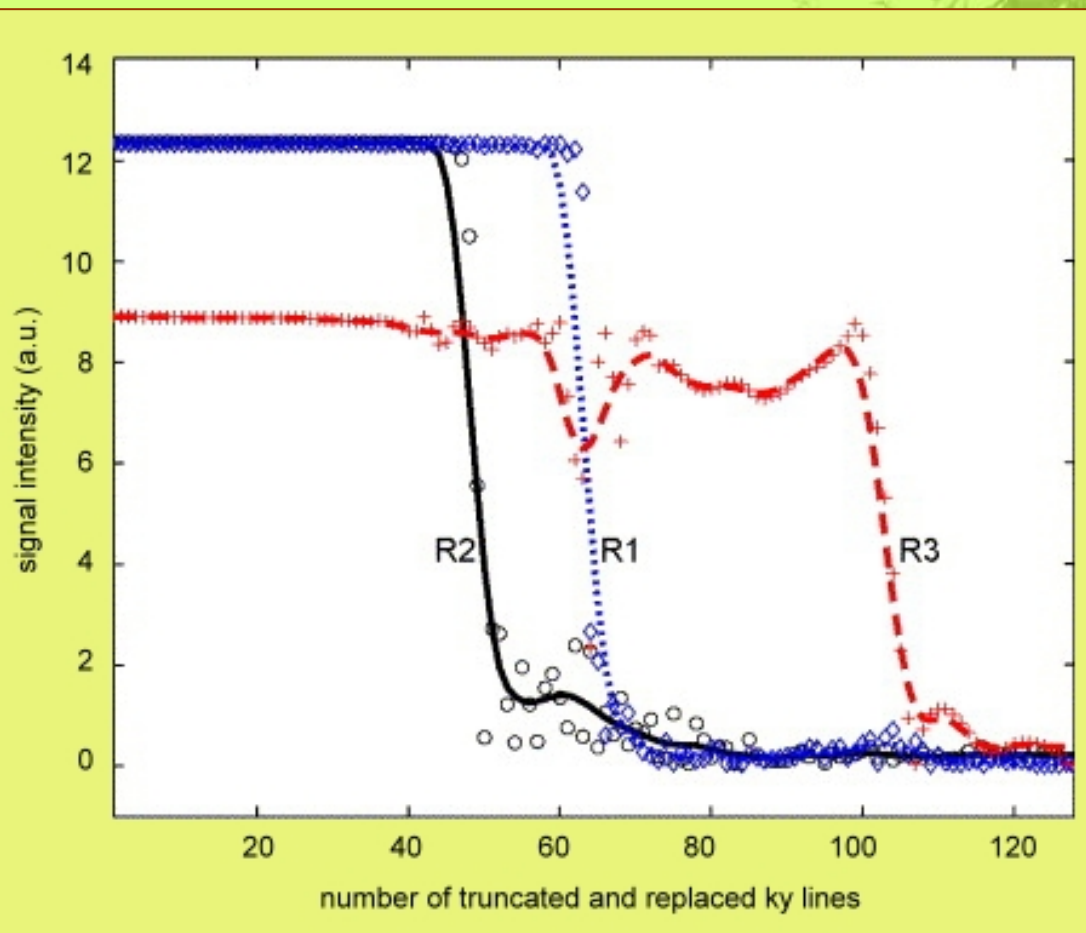


pixel by pixel

$$\Delta ky_{R1}=0$$

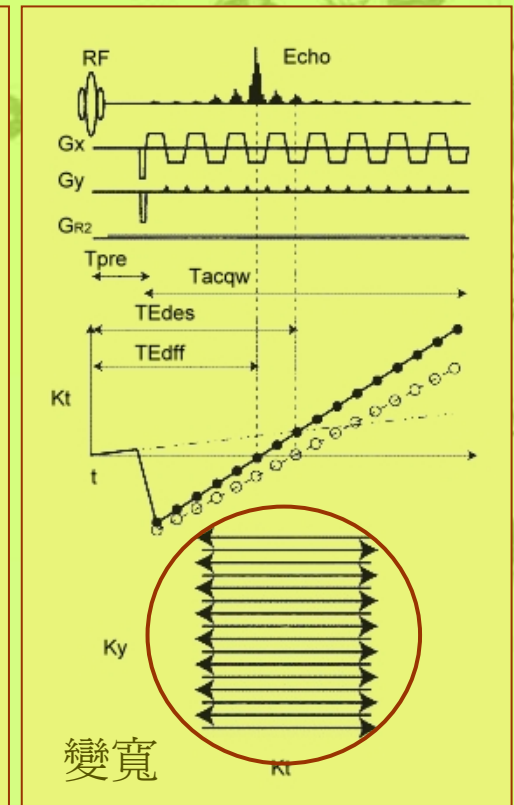
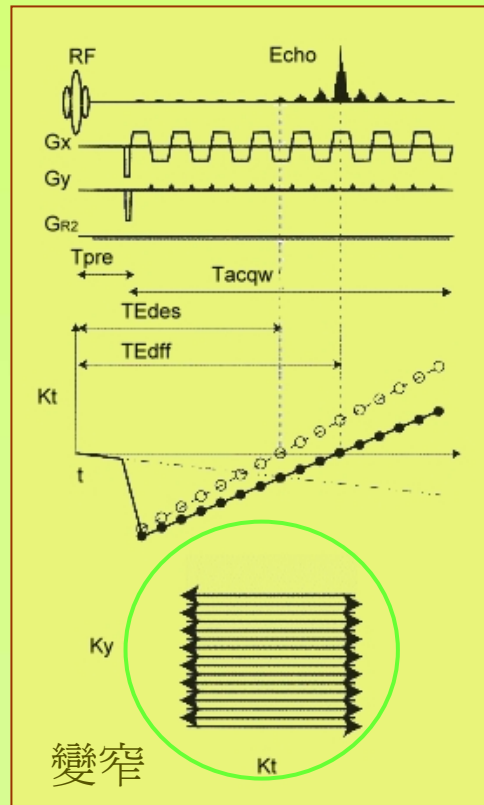
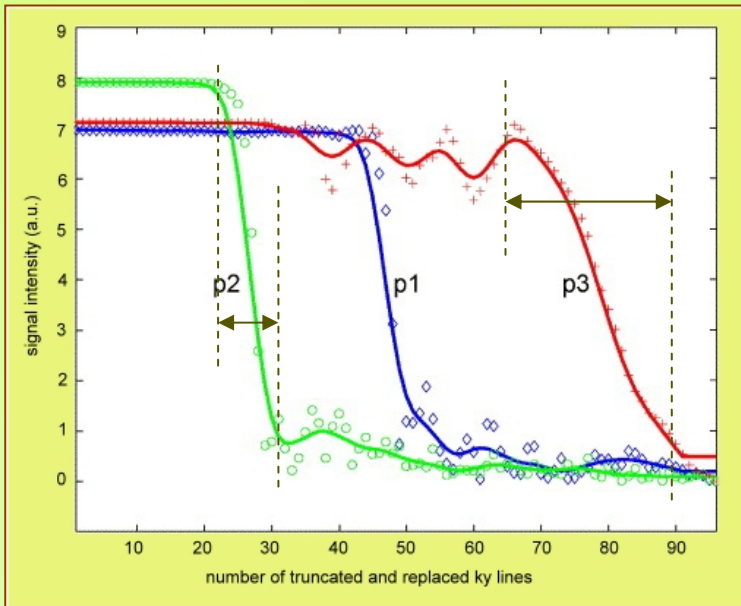
$$\Delta ky_{R2}=-16$$

$$\Delta ky_{R3}=39$$



Transition Width

- transition band 的寬度不同
- 和gradient有關

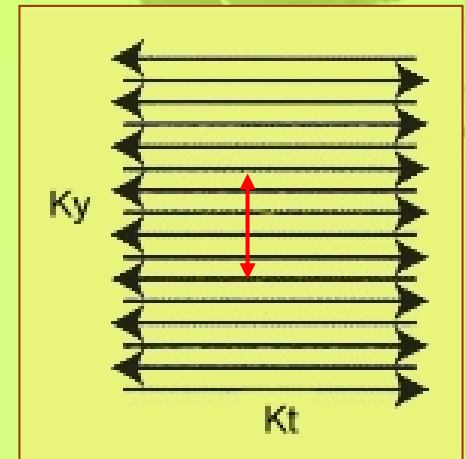
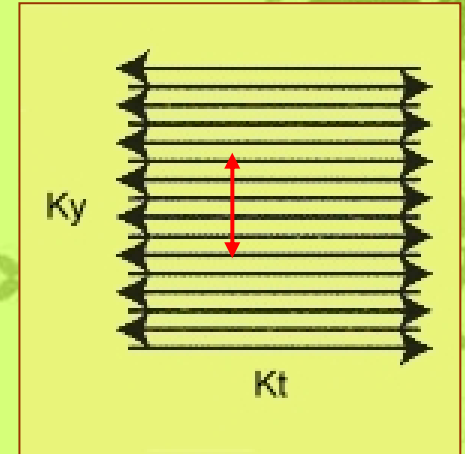


$$\delta\tilde{k}_y = \delta k_y + \gamma G_{R2} T_{esp} \quad \text{ky lines 之間の間隔}$$

$$= \frac{1}{\text{FOV}_y} + \gamma G_{R2} T_{esp}$$

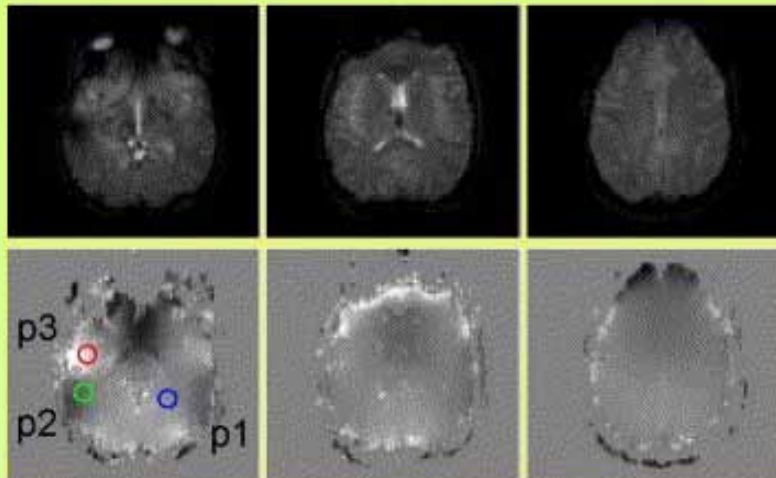
$$E = n\delta k_y = m\delta\tilde{k}_y \quad E = \text{width of k-space peak}$$

$$R = \frac{m}{n} = \frac{1}{1 + \gamma G_{R2} T_{esp} \text{FOV}_y}$$

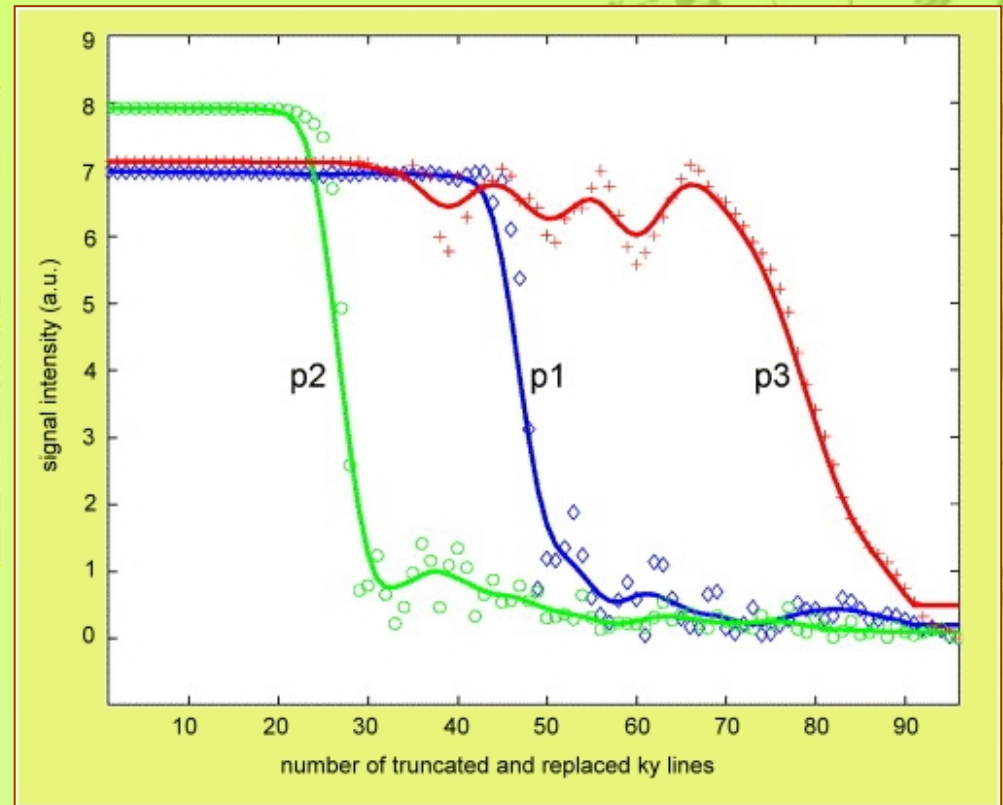


Displacement Map

EPI image




displacement map





Field Mapping

- displacement map \rightarrow gradient
 - 用前面的公式
 - gradient $\rightarrow \Delta B_0$
 - 並非全部線性變化
 - 用類似積分的方法
- 

Field Mapping

$$\Delta B_0(x_0, y_0 + \delta y)$$

$$\approx \Delta B_0(x_0, y_0) + \left[\delta y \frac{G_y(x_0, y_0) + G_y(x_0, y_0 + \delta y)}{2} \right]$$

$$\Delta B_0(x_0 + \delta x, y_0)$$

$$\approx \Delta B_0(x_0, y_0) + \left[\delta x \frac{G_x(x_0, y_0) + G_x(x_0 + \delta x, y_0)}{2} \right]$$

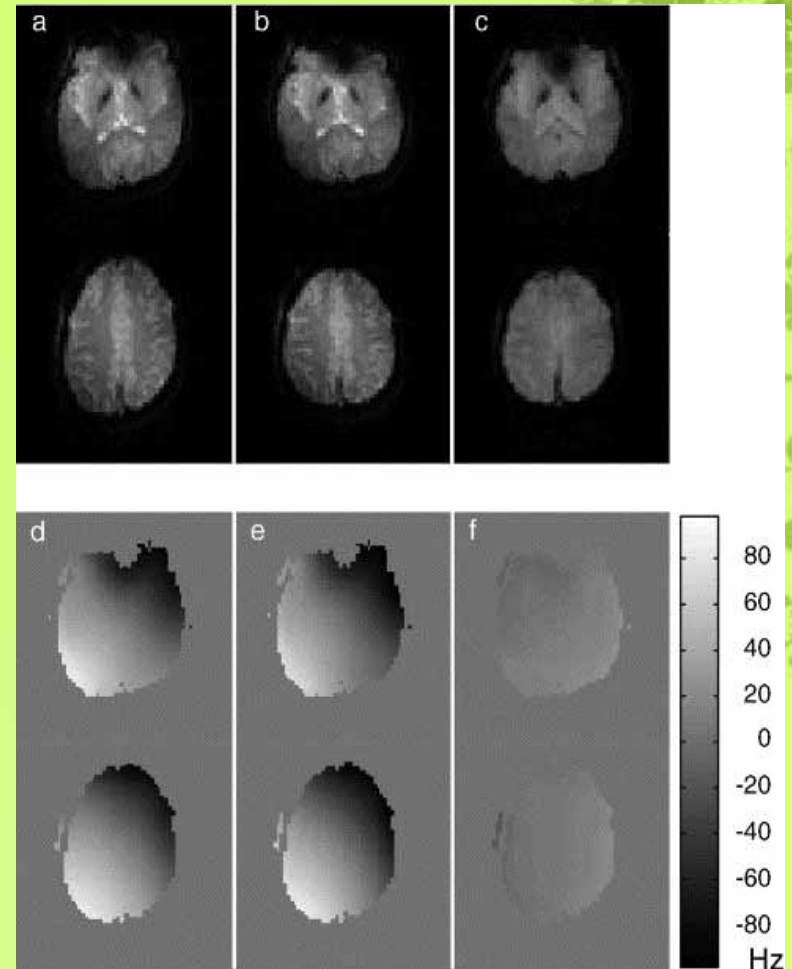
- Offset of B_0
 - Avg of $B_0 = 0$

correction

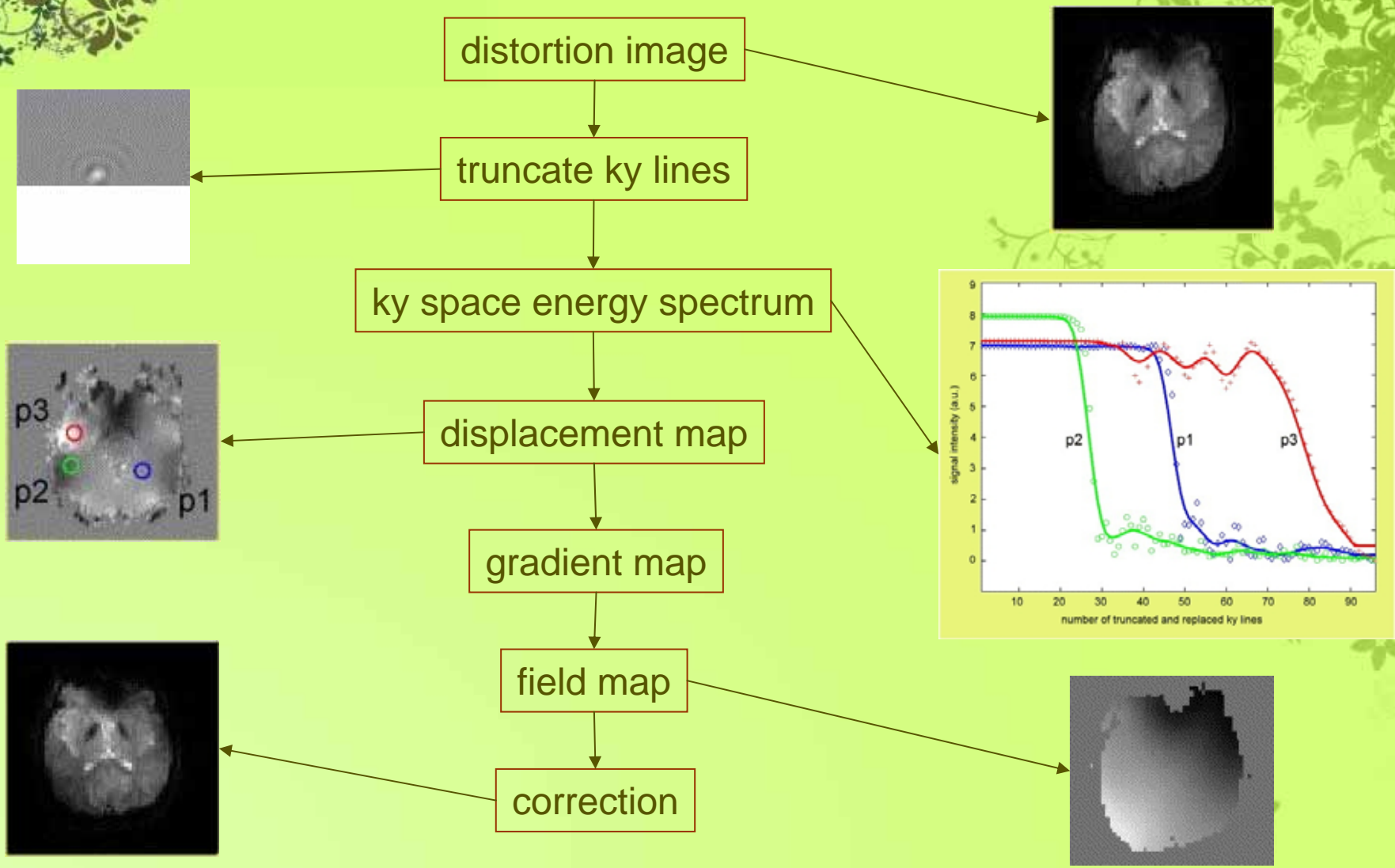
$$\Delta y = N_y \Delta B_0(x, y) N_y T_{esp}$$

$$y = \tilde{y} - \Delta y$$

- 決定 y
- 計算 y' 的位置
- 把distortion y 的值填到正確的 y

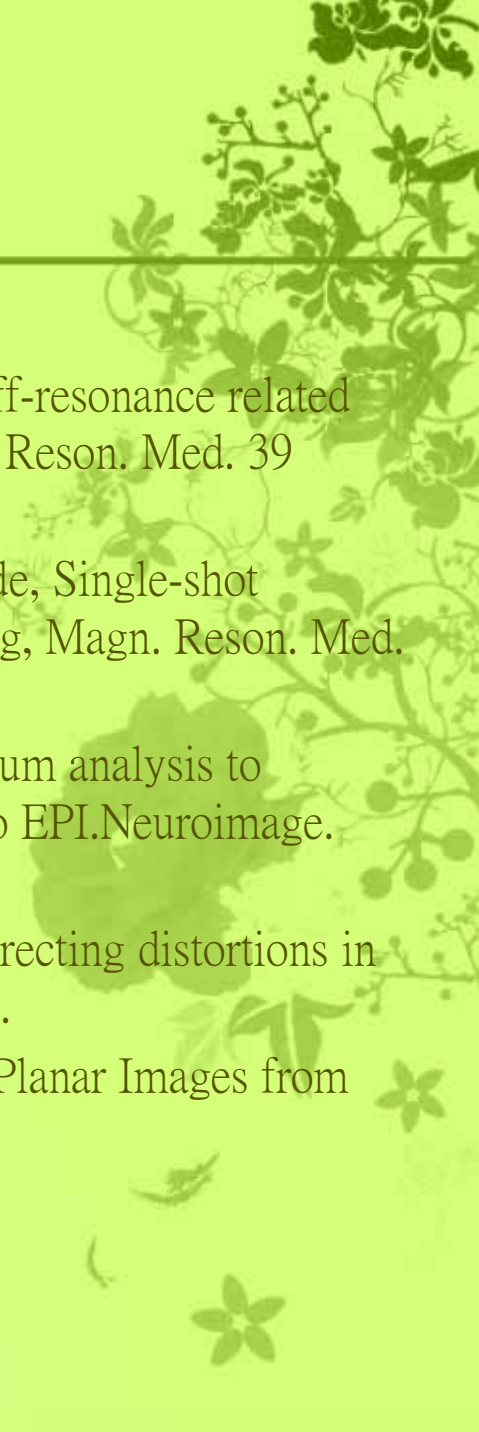


Flow Chart





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- Thanks for Your Attention



- THE END