Shinnar-Le Roux Transform

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2009/10/26

Shinnar-Le Roux transform

- 簡稱為**SLR** transform
- 想要什麼樣的slice profile, 可以反推回去其RF pulse
- 可以不必考慮傅立葉轉換中, Bloch equations 所帶來的非線性問題

+

= SLR

Section I

Nonlinearity of Bloch Equation

Fourier Transform Approximation

- defining the transverse magnetization:

$$M_{\perp}(t) = i\gamma e^{-i\triangle\omega t}\int_0^t M_z(t^{'})B_1(t^{'})e^{i\triangle\omega t^{'}}dt^{'}$$

Fourier Transform Approximation

- 再看一次…

$$M_{\perp}(t) = i\gamma e^{-i\triangle\omega t}\int_0^t M_z(t')B_1(t')e^{i\triangle\omega t'}dt'$$

- 是inverse Fourier transform啊..

$$M_{\perp} \propto FT^{-1}\left[M_z(t)B_1(t)\right]$$

Fourier Transform Approximation

- 看的更仔細一點..

$$M_{\perp}(t) = i\gamma e^{-i\triangle\omega t}\int_0^t M_z(t')B_1(t')e^{i\triangle\omega t'}dt'$$

- $M_z(t)$是隨著RF pulse變化的,根本是無從得知...

Fourier Transform Approximation

- To a good approximation, a small flip angles, $M_z(t) \approx M_0$

$$M_{\perp}(t) \approx i\gamma M_0 e^{-i\triangle\omega t}\int_0^t B_1(t')e^{i\triangle\omega t'}dt'$$

Fourier Transform Approximation

- $M_z(t) \approx M_0$,那麼θ是要多小呢?

- θ< 30° : 不錯
- 30° <θ<90° : 勉強接受
- θ> 90° : 那就慘了

前情提要

- 上週:John Pauly et al
- 本週:John Pauly et al

- 上週:小角度+δ function, 讓傅立葉轉換復活
- 本週:矩陣取代傅立葉轉換

Section II

Just rotate it

3x3 orthogonal matrix

- Bloch Equation

$$\frac{d}{dt}\begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} = \gamma \begin{bmatrix} 0 & G_x & -B_{1,y} \\ -G_x & 0 & B_{1,x} \\ B_{1,y} & -B_{1,x} & 0 \end{bmatrix} \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix}$$

$$\frac{d}{dt}\begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} = -\gamma \left(B_{1,x} \boxed{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}} + B_{1,y} \boxed{\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}} + Gx \boxed{\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}} \right) \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix}$$

S_x S_y S_z

3x3 orthogonal matrix

- Define:

$$\underline{M} = \left[M_x, M_y, M_z \right]^T$$

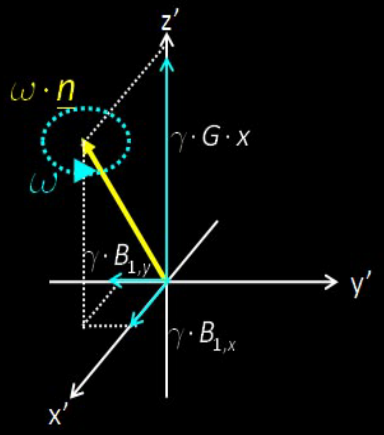
$$\underline{S} = \left(S_x, S_y, S_z \right)$$

$$\boxed{\frac{d}{dt}\underline{M} = \left[\left(-\gamma \cdot B_{1,x}, -\gamma \cdot B_{1,y}, -\gamma \cdot G \cdot x \right) \cdot \underline{S} \right] \underline{M}}$$

旋轉角速度

旋轉軸心

$$\begin{cases} \omega = -\gamma \sqrt{\left(B_{1,x}\right)^2 + \left(B_{1,y}\right)^2 + (Gx)^2} \\ \underline{n} = \dfrac{\gamma}{|\omega|} \left(B_{1,x}, B_{1,y}, Gx \right) \end{cases}$$



- 化簡

$$\frac{d}{dt}\underline{M} = \omega\left(\underline{n},\underline{s}\right)\underline{M}$$

- 解個一階常微分方程式

$$R = e^{(\underline{n},\underline{s})\int_{-\infty}^{t}\omega(\tau)d\tau}, \text{ general solution} \equiv R = e^{(\underline{n},\underline{s})\theta}$$

$$M^{+} = RM^{-}$$

3x3 orthogonal matrix

- 3x3 matrices are tedious!
- How about 2x2 matrices?

Spinor Plane

- The real orthogonal transformations in 3D space correspond to unitary transformations in the spinor space

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} z & x - iy \\ x + iy & -z \end{bmatrix}$$

Quantum Mechanical

- Pauli spin matrices:

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \ \sigma_{\mathrm{y}} = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \ \sigma_{\mathrm{z}} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$x\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + y\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} + z\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} z & x - iy \\ x + iy & -z \end{bmatrix}$$

Quantum Mechanical

- Spinor: the elements of a complex vector space introduced to the notion of spatial vector
- $\psi_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$
- $M_x = \psi^* \sigma_x \psi,\ M_y = \psi^* \sigma_y \psi,\ M_z = \psi^* \sigma_z \psi$

2x2 unitary matrix

- Differential equation corresponds to the Bloch Equation:

$$\psi^{+} = Q\psi$$

$$Q = I\cos\frac{\phi}{2} - i\left(\underline{N}, \underline{\sigma}\right)\sin\frac{\phi}{2}$$

$$Q = \underbrace{\boxed{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}}_{\mathbf{I}}\cos\frac{\phi}{2} + \underbrace{\boxed{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}}_{\sigma_x}\left(-iN_x\sin\frac{\phi}{2}\right) + \underbrace{\boxed{\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}}}_{\sigma_y}\left(-iN_y\sin\frac{\phi}{2}\right) + \underbrace{\boxed{\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}}}_{\sigma_z}\left(-iN_z\sin\frac{\phi}{2}\right)$$

2x2 unitary matrix

- Final,

$$Q = \begin{bmatrix} \cos\frac{\phi}{2} - iN_z \sin\frac{\phi}{2} & -i\left(N_x - iN_y\right)\sin\frac{\phi}{2} \\ -i\left(N_x + iN_y\right)\sin\frac{\phi}{2} & \cos\frac{\phi}{2} + iN_z \sin\frac{\phi}{2} \end{bmatrix}$$

2x2 unitary matrix

- Define:

$$\begin{cases} \alpha = \cos\frac{\phi}{2} - i\widehat{N}_z \sin\frac{\phi}{2} \\ \beta = -i\left(\widehat{N}_x + i\widehat{N}_y\right)\sin\frac{\phi}{2} \end{cases} \qquad \alpha\alpha^* + \beta\beta^* = 1$$

α and β are the Cayley-Klein parameters

- Then,

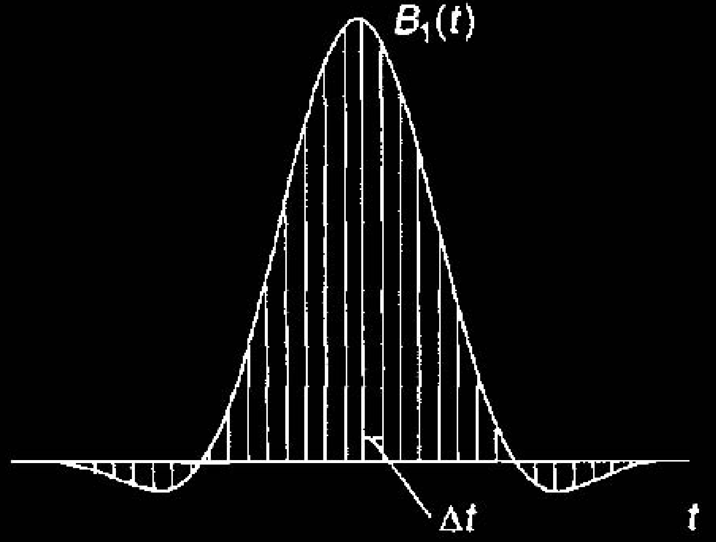
$$Q = \begin{bmatrix} \cos\frac{\phi}{2} - iN_z \sin\frac{\phi}{2} & -i\left(N_x - iN_y\right)\sin\frac{\phi}{2} \\ -i\left(N_x + iN_y\right)\sin\frac{\phi}{2} & \cos\frac{\phi}{2} + iN_z \sin\frac{\phi}{2} \end{bmatrix} \Rightarrow Q = \begin{bmatrix} \alpha & -\beta^* \\ \beta & \alpha^* \end{bmatrix}$$

Section III

Matrix in the Spin Domain

Hard Pulse Approximation

- A soft pulse, $B_1(t)$, can be approximated by n short hard pulses:



Spin Domain Bloch Equation

- Bloch equation in the matrix representation:

$$\begin{bmatrix} M_x' \\ M_y' \\ M_z' \end{bmatrix} = \gamma \begin{bmatrix} 0 & G_x & -B_{1,y} \\ -G_x & 0 & B_{1,x} \\ B_{1,y} & -B_{1,x} & 0 \end{bmatrix} \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix}$$

$$M(T) = RM(0)$$

- Total rotation matrices:

$$R = R_n R_{n-1} \cdots R_1$$

Spin Domain Representation

- rotation parameters:

  --rotation angle

$$\phi_j = -\gamma \triangle t \sqrt{\left|\vec{B}_{1,j}\right|^2 + (Gx)^2}$$

  --rotation axis

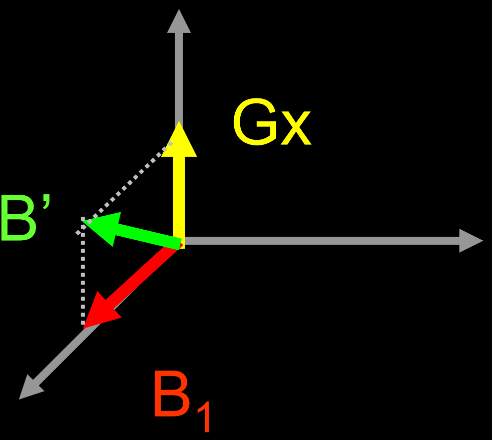
$$\hat{N}_j = \frac{\gamma \triangle t}{\left|\phi_j\right|}\left(\vec{B}_{1,j,x}, \vec{B}_{1,j,y}, Gx\right)$$

$\vec{B}_{1,j}$: the i-th magnetic filed

$\triangle$t: the turn-on duration of $\vec{B}_{1,j}$

G: the gradient strength

x: the position



Spin Domain Representation

- There rotations can also be represented by 2×2 unitary matrices:

$$Q = \begin{bmatrix} \alpha & -\beta^* \\ \beta & \alpha^* \end{bmatrix}$$

- Cayley-Klein parameters

$$\alpha = \cos\frac{\phi}{2} - i\widehat{N}_z \sin\frac{\phi}{2}$$

$$\beta = -i\left(\widehat{N}_x + i\widehat{N}_y\right)\sin\frac{\phi}{2}$$

$$\alpha\alpha^* + \beta\beta^* = 1$$

Spin Domain Representation

- Cayley-Klein parameters for the *j*th interval:

$$a_j = \cos\frac{\phi_j}{2} - i\widehat{N}_{z,j}\sin\frac{\phi_j}{2}$$

$$b_j = -i\left(\widehat{N}_{x,j} + i\widehat{N}_{y,j}\right)\sin\frac{\phi_j}{2}$$

Spin Domain Representation

- 每一次小轉動

$$Q_j = \begin{bmatrix} a_j & -b_j^* \\ b_j & a_j^* \end{bmatrix}$$

- 再合成大轉動

$$Q = Q_n Q_{n-1} \cdots Q_1$$

Spin Domain Representation

- 依樣畫葫蘆

$$\begin{bmatrix} \alpha_n & -\beta_n^* \\ \beta_n & \alpha_n^* \end{bmatrix}$$

$$= \begin{bmatrix} a_n & -b_n^* \\ b_n & a_n^* \end{bmatrix} \boxed{\begin{bmatrix} a_{n-1} & -b_{n-1}^* \\ b_{n-1} & a_{n-1}^* \end{bmatrix} \cdots \begin{bmatrix} a_0 & -b_0^* \\ b_0 & a_0^* \end{bmatrix}}$$

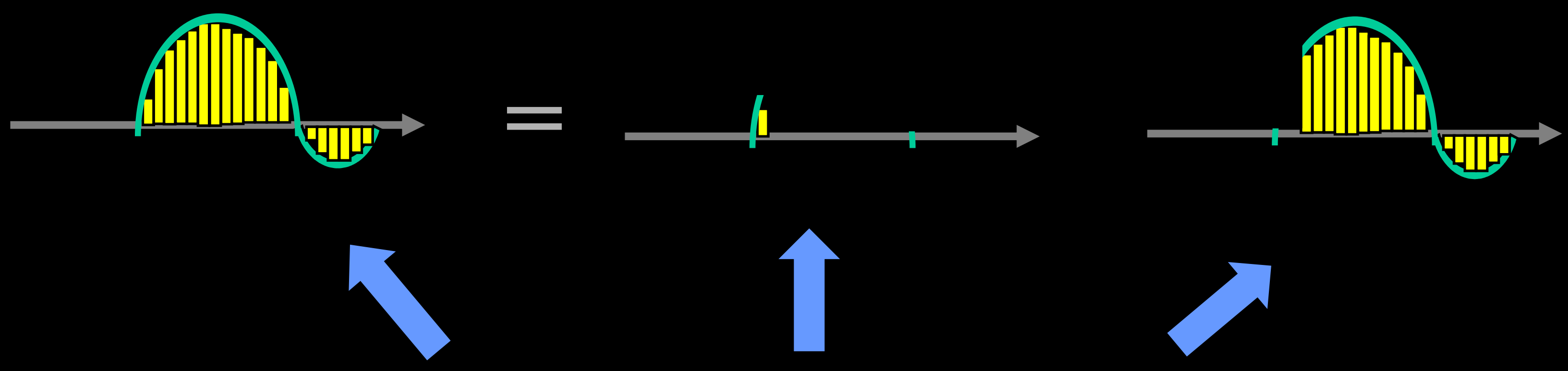
$$\begin{bmatrix} \alpha_{n-1} & -\beta_{n-1}^* \\ \beta_{n-1} & \alpha_{n-1}^* \end{bmatrix}$$

- 又是a和b, 又是α和β的...............

- a和b: 每個被分開的小轉動
- α和β :一堆a和b合成起來的大轉動

$$\begin{bmatrix} \alpha_j \\ \beta_j \end{bmatrix} = \begin{bmatrix} a_j & -b_j^* \\ b_j & a_j^* \end{bmatrix} \begin{bmatrix} \alpha_{j-1} \\ \beta_{j-1} \end{bmatrix}$$

畫畫



$$\begin{bmatrix} \alpha_j \\ \beta_j \end{bmatrix} = \begin{bmatrix} a_j & -b_j^* \\ b_j & a_j^* \end{bmatrix} \begin{bmatrix} \alpha_{j-1} \\ \beta_{j-1} \end{bmatrix}$$

Spin Domain Representation

- No rotation as the initial condition

$$\psi_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \psi_1 = \begin{bmatrix} \alpha_1 & -\beta_1^* \\ \beta_1 & \alpha_1^* \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \beta_1 \end{bmatrix}$$

- The product of two unitary matrices is also unitary

$$\begin{bmatrix} \alpha_j \\ \beta_j \end{bmatrix} = \begin{bmatrix} a_j & -b_j^* \\ b_j & a_j^* \end{bmatrix} \begin{bmatrix} \alpha_{j-1} \\ \beta_{j-1} \end{bmatrix}$$

最後還是回到MR

$$M_x = \psi^* \sigma_x \psi,\ M_y = \psi^* \sigma_y \psi,\ M_z = \psi^* \sigma_z \psi$$

$$\psi = \begin{pmatrix} \alpha & -\beta^* \\ \beta & \alpha^* \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$M_z = \begin{pmatrix} \alpha^* & \beta^* \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \alpha^* & \beta^* \end{pmatrix} \begin{pmatrix} \alpha \\ -\beta \end{pmatrix} = \alpha\alpha^* - \beta\beta^*$$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

不用解Bloch Equation

$$M_{xy} = M_x + iM_y$$

$$\begin{pmatrix} M_{xy}^{+} \\ M_{xy}^{+*} \\ M_{z}^{+} \end{pmatrix} = \begin{pmatrix} \left(\alpha^*\right)^2 & -\beta^2 & 2\alpha^*\beta \\ -\left(\beta^*\right)^2 & \alpha^2 & 2\alpha\beta^* \\ -\alpha^*\beta^* & -\alpha\beta & \alpha\alpha^* - \beta\beta^* \end{pmatrix} \begin{pmatrix} M_{xy}^{-} \\ M_{xy}^{-*} \\ M_{z}^{-} \end{pmatrix}$$

只要求出α和β，就可以求出全部的磁矩

整理一下

- **2x2 unitary matrix:**
  --spinor
  --rotation matrix:Q
  -- ψ^{+}=Q ψ^{-}

- **3x3 orthogonal matrix:**
  --magnetization
  --rotation matrix: R
  --$M^{+}=RM^{-}$

Section IV

Shinnar-Le Roux Transform

SLR

- **Forward SLR**

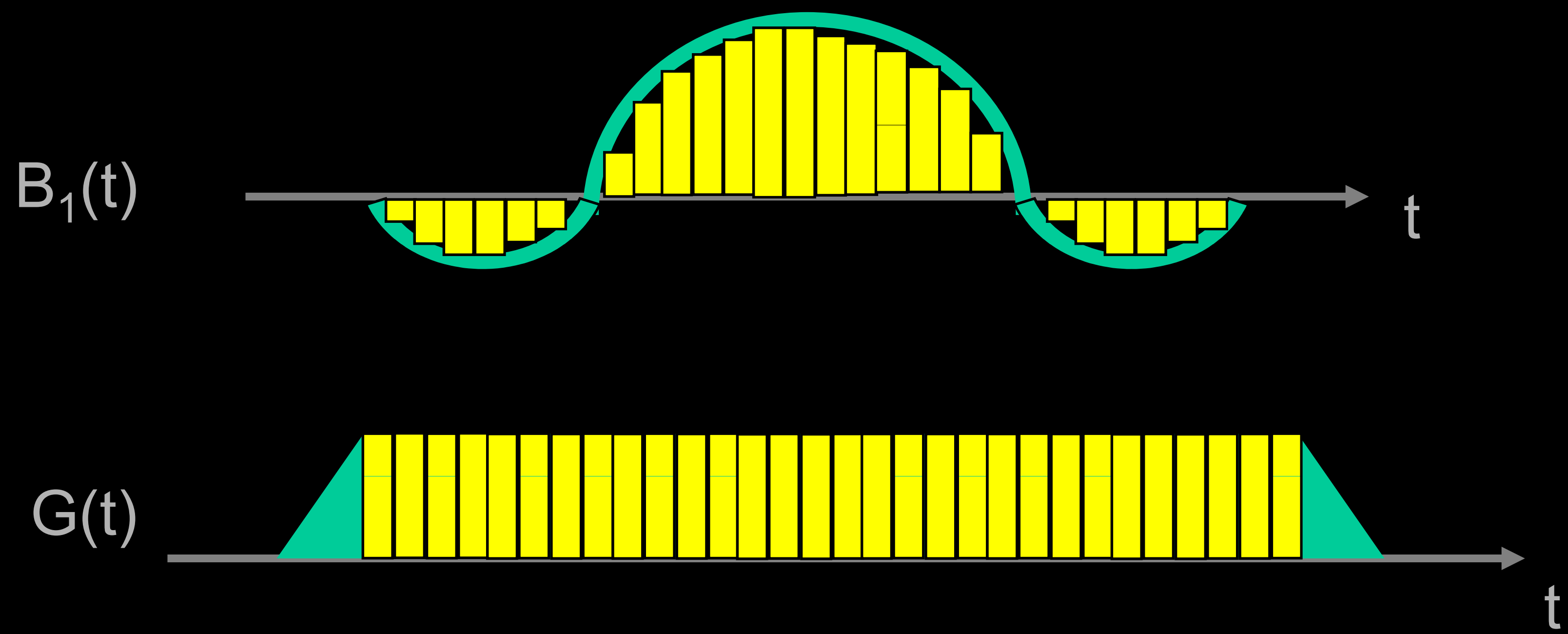
  --給定一個已知的外加磁場, 可以求得其對應的slice profile

- **Inverse SLR**

  --腦袋裡已經有一個想要的slice profile, 反推回去其轉動矩陣, 再求得對應的外加磁場

Forward SLR

- 除了RF 所帶來的角度改變$-\gamma B_1 \triangle t$, 還有梯度磁場所帶來的角度改變$-\gamma Gx \triangle t$



Forward SLR

- Hard pulse :

$$Q_{RF} = \begin{bmatrix} C_j & -S_j^* \\ S_j & C_j^* \end{bmatrix}$$

$$C_j = \cos\left(\frac{\gamma \left|B_1(t_j)\right| \triangle t}{2}\right)$$

$$S_j = ie^{i\angle B_1(t_j)} \sin\left(\frac{\gamma \left|B_1(t_j)\right| \triangle t}{2}\right)$$

$$\alpha = \cos\frac{\phi}{2} - i\widehat{N}_z \sin\frac{\phi}{2}$$

0

$$\beta = -i\underbrace{\left(\widehat{N}_x + i\widehat{N}_y\right)}_{e^{i\angle B_1(t_j)}}\sin\frac{\phi}{2}$$

Forward SLR

- Gradient :

$$Q_{gradient} = \begin{bmatrix} z^{1/2} & 0 \\ 0 & z^{-1/2} \end{bmatrix} = z^{1/2} \begin{bmatrix} 1 & 0 \\ 0 & z^{-1} \end{bmatrix}$$

$$\alpha = \cos\frac{\phi}{2} - i\widehat{N}_z \sin\frac{\phi}{2} = e^{-i\phi/2} \qquad , z = e^{i\phi} = e^{i\gamma G x \Delta t}$$

$$\beta = -i\left(\widehat{N}_x + i\widehat{N}_y\right)\sin\frac{\phi}{2} = 0$$

0

Forward SLR

- The total rotation matrix:

$$Q_j = \begin{bmatrix} C_j & -S_j^* \\ S_j & C_j^* \end{bmatrix} \begin{bmatrix} z^{1/2} & 0 \\ 0 & z^{-1/2} \end{bmatrix}$$

$$= \begin{bmatrix} C_j & -S_j^* \\ S_j & C_j \end{bmatrix} z^{1/2} \begin{bmatrix} 1 & 0 \\ 0 & z^{-1} \end{bmatrix}$$

$$C_j = \cos\left(\frac{\gamma \left|B_1(t_j)\right| \Delta t}{2}\right) = 實數$$

Forward SLR

- If we substitute into the precession:

$$\begin{bmatrix} \alpha_j \\ \beta_j \end{bmatrix} = z^{1/2} \begin{bmatrix} C_j & -S_j^* \\ S_j & C_j \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & z^{-1} \end{bmatrix} \begin{bmatrix} \alpha_{j-1} \\ \beta_{j-1} \end{bmatrix}$$

- Get rid of half power of z, define:

$$\begin{bmatrix} A_j \\ B_j \end{bmatrix} = z^{j/2} \begin{bmatrix} \alpha_j \\ \beta_j \end{bmatrix}$$

代入

Forward SLR

- then:

$$\begin{bmatrix} A_j \\ B_j \end{bmatrix} = \begin{bmatrix} C_j & -S_j^* \\ S_j & C_j \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & z^{-1} \end{bmatrix} \begin{bmatrix} A_{j-1} \\ B_{j-1} \end{bmatrix}$$

- The initial state is no rotation, so:

$$\begin{bmatrix} A_0 \\ B_0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Forward SLR

- final,

$$\begin{bmatrix} A_1 \\ B_1 \end{bmatrix} = \begin{bmatrix} C_1 \\ S_1 \end{bmatrix}$$

$$\begin{bmatrix} A_2 \\ B_2 \end{bmatrix} = \begin{bmatrix} C_1 C_2 - S_1 S_2^* z^{-1} \\ S_2 C_1 + S_1 C_2 z^{-1} \end{bmatrix}$$

$$\begin{bmatrix} A_3 \\ B_3 \end{bmatrix} = \begin{bmatrix} C_3 C_2 C_1 - \left(C_3 S_2^* S_1 + S_3^* S_2 C_1 \right) z^{-1} - S_3^* C_2 S_1 z^{-2} \\ S_3 C_2 C_1 - \left(S_3 S_2^* S_1 - C_3 S_2 C_1 \right) z^{-1} + C_3 C_2 S_1 z^{-2} \end{bmatrix}$$

$A_j(z)$ and $B_j(z)$ are each polynomials of order $j-1$ in the variable z^{-1}

Forward SLR

- Define:

$$A_n(z) = \sum_{j=0}^{n-1} a_j z^{-j}$$

$$B_n(z) = \sum_{j=0}^{n-1} b_j z^{-j}$$

j-1 is lower order than j

- 但是$\alpha_{n,}$ β_n 才是真正的轉動元素

$$\begin{bmatrix} A_j \\ B_j \end{bmatrix} = z^{j/2} \begin{bmatrix} \alpha_j \\ \beta_j \end{bmatrix} \Leftrightarrow z^{-j/2} \begin{bmatrix} A_j \\ B_j \end{bmatrix} = \begin{bmatrix} \alpha_j \\ \beta_j \end{bmatrix}$$

Forward SLR

- 整理一下

$$A_n(z) = \sum_{j=0}^{n-1} a_j z^{-j} \qquad \Leftrightarrow \qquad \alpha_n(z) = z^{-n/2}\left(\sum_{j=0}^{n-1} a_j z^{-j}\right)$$

$$B_n(z) = \sum_{j=0}^{n-1} b_j z^{-j} \qquad \qquad \beta_n(z) = z^{-n/2}\left(\sum_{j=0}^{n-1} b_j z^{-j}\right)$$

Forward SLR

- Forward SLR:

  RF + spatial gradient = two polynomials
- Get the RF pulse → get the slice profile

- 休息是為了看更多的數學
- 所以休息一下吧

Inversion SLR

- Given two polynomials → producing the B1(t)
- Design A_n and B_n, design $B_1(t)$

Inversion SLR

- 先看回forward SLR

$$\begin{bmatrix} A_j \\ B_j \end{bmatrix} = \begin{bmatrix} C_j & -S_j^* \\ S_j & C_j \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & z^{-1} \end{bmatrix} \begin{bmatrix} A_{j-1} \\ B_{j-1} \end{bmatrix}$$

- 想要求得A_{i-1}, B_{i-1}:

$$\begin{bmatrix} A_{j-1} \\ B_{j-1} \end{bmatrix} = \left(\begin{bmatrix} C_j & -S_j^* \\ S_j & C_j \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & z^{-1} \end{bmatrix} \right)^{-1} \begin{bmatrix} A_j \\ B_j \end{bmatrix}$$

$$= \begin{bmatrix} C_j & S_j^* \\ -S_j z & C_j z \end{bmatrix} \begin{bmatrix} A_j \\ B_j \end{bmatrix}$$

Inversion SLR

$$\begin{bmatrix} A_{j-1} \\ B_{j-1} \end{bmatrix} = \begin{bmatrix} C_j & S_j^* \\ -S_j z & C_j z \end{bmatrix} \begin{bmatrix} A_j \\ B_j \end{bmatrix} = \begin{bmatrix} C_j A_j + S_j^* B_j \\ z\left(-S_j A_j + C_j B_j\right) \end{bmatrix}$$

- $[A_{j-1}\ B_{j-1}]^T$ are lower order than $[A_j\ B_j]^T$
  --the highest term of A_{j-1}
  --the lowest term of B_{j-1}

  $\Rightarrow$ drop out

Inversion SLR

$$\begin{bmatrix} A_{j-1} \\ B_{j-1} \end{bmatrix} = \begin{bmatrix} C_j A_j + S_j^* B_j \\ z\left(-S_j A_j + C_j B_j\right) \end{bmatrix}$$

$$\begin{bmatrix} A_1 \\ B_1 \end{bmatrix} = \begin{bmatrix} C_1 \\ S_1 \end{bmatrix}$$

$$\begin{bmatrix} A_2 \\ B_2 \end{bmatrix} = \begin{bmatrix} C_1 C_2 - S_1 S_2^* z^{-1} \\ S_2 C_1 + S_1 C_2 z^{-1} \end{bmatrix}$$

$$\begin{bmatrix} A_3 \\ B_3 \end{bmatrix} = \begin{bmatrix} C_3 C_2 C_1 - \left(C_3 S_2^* S_1 + S_3^* S_2 C_1\right) z^{-1} - S_3^* C_2 S_1 z^{-2} \\ S_3 C_2 C_1 - \left(S_3 S_2^* S_1 - C_3 S_2 C_1\right) z^{-1} + C_3 C_2 S_1 z^{-2} \end{bmatrix}$$

Inversion SLR

$$C_j A_{j,j-1} + S_j^* B_{j,j-1} = 0$$

$$-S_j A_{j,0} + C_j B_{j,0} = 0$$

$A_{j,m}$: the mth term of the polynimail A_j

$B_{j,m}$: the mth term of the polynimail B_j

- Choosing the low order recursion:

$$-S_j A_{j,0} + C_j B_{j,0} = 0$$

$$\frac{B_{j,0}}{A_{j,0}} = \frac{S_j}{C_j} = \frac{ie^{i\theta_j} \sin {\phi_j}/{2}}{\cos {\phi_j}/{2}}$$

Inversion SLR

- The tip angle produced by jth of the pulse:

$$\phi_j = 2\tan^{-1}\left|\frac{B_{j,0}}{A_{j,0}}\right|$$

- The phase of the RF pulse:

$$\theta_j = \angle\left(\frac{-iB_{j,0}}{A_{j,0}}\right)$$

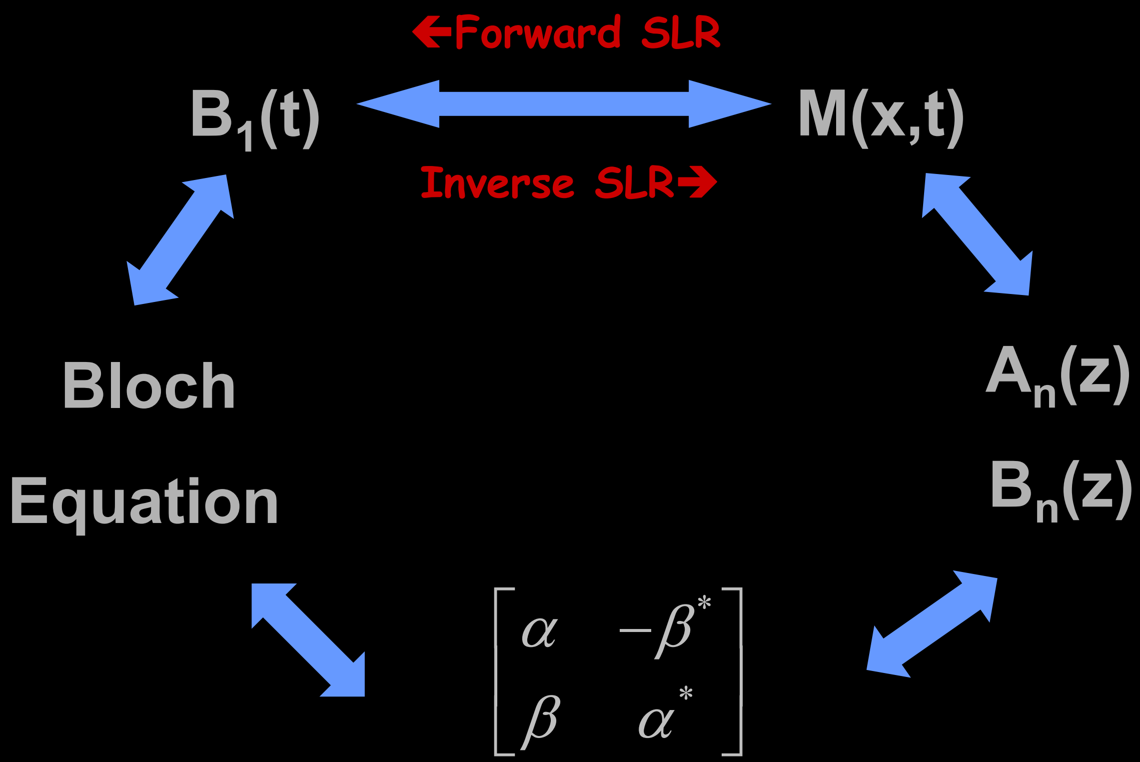
$$\frac{B_{j,0}}{A_{j,0}} = \frac{S_j}{C_j} = \frac{ie^{i\angle B_1(t_j)}\sin\left(\frac{\gamma\left|B_1(t_j)\right|\triangle t}{2}\right)}{\cos\left(\frac{\gamma\left|B_1(t_j)\right|\triangle t}{2}\right)}$$

Inversion SLR

- The waveform of the radiofrequency complex envelope:

$$B_{1,j} = \frac{1}{\gamma \vartriangle t} \phi_j e^{i\theta_j}$$

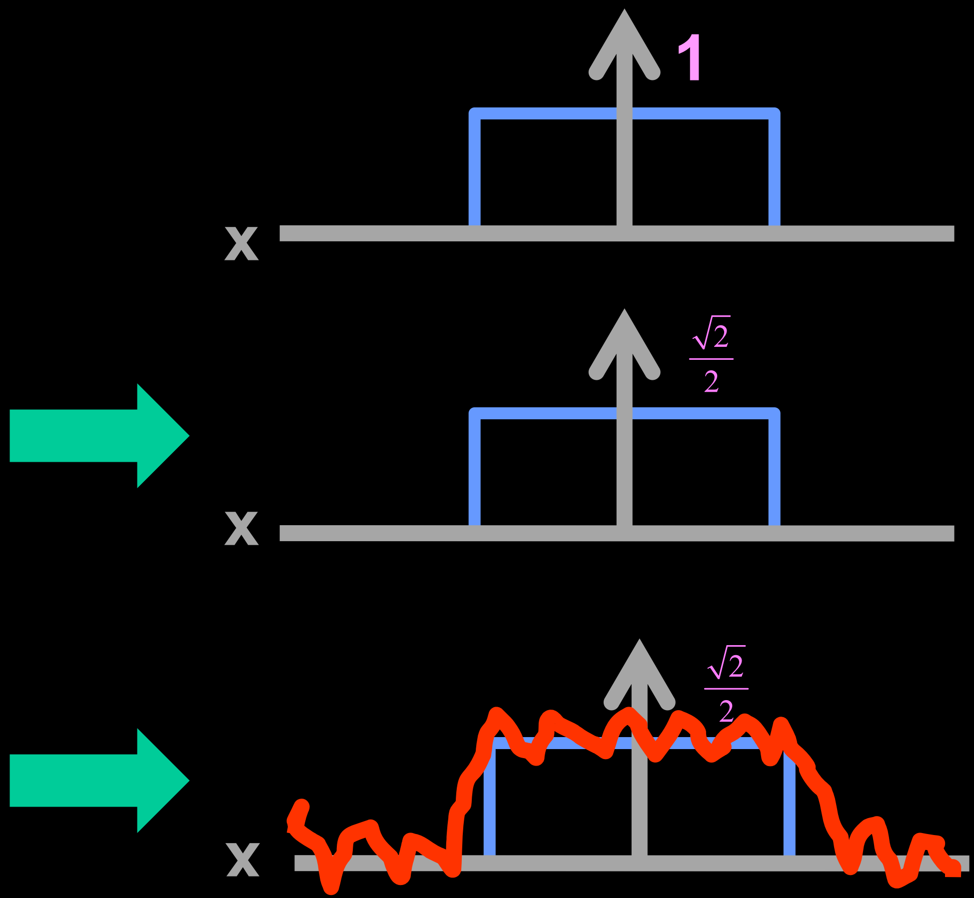
SLR Algorithm



Section V

RF pulse design with SLR

RF Pulse Design



Ideal slice profile

Ideal B_I profile

$$B_I = i\left(n_x + n_y\right)\sin\frac{\phi(x)}{2} = i\sin\frac{\phi(x)}{2}$$

$$\begin{Bmatrix} \text{in slice: } i\sin\frac{\pi}{4} = i\frac{\sqrt{2}}{2} \\ \text{out slice: } i\sin 0 = 0 \end{Bmatrix}$$

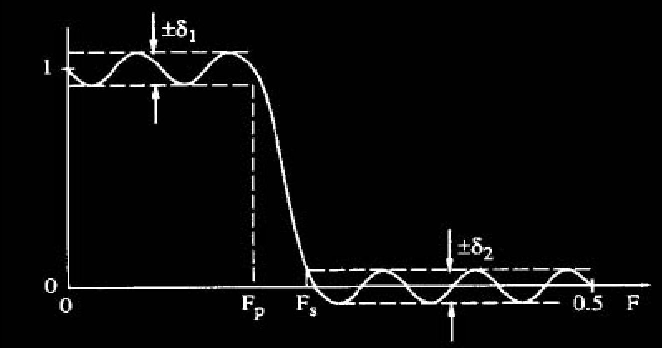
Approximate ideal B_I profile, B_n

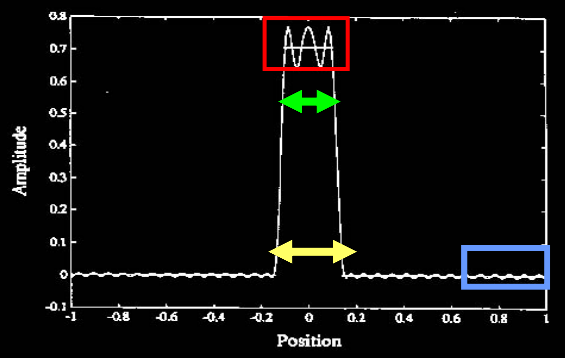
RF pulse design

1.PM FIR digital filter 製造出 B_{n},

--PM: Parks-McClellan algorithm

--FIR: finite impulse response





δ_1 : the amplitude of the passband ripple

δ_2 : the amplitude of the stopband ripple

F_p : the passband edge

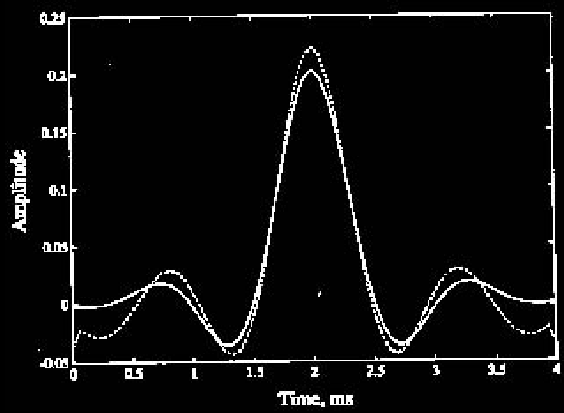
F_s : the stopband edge

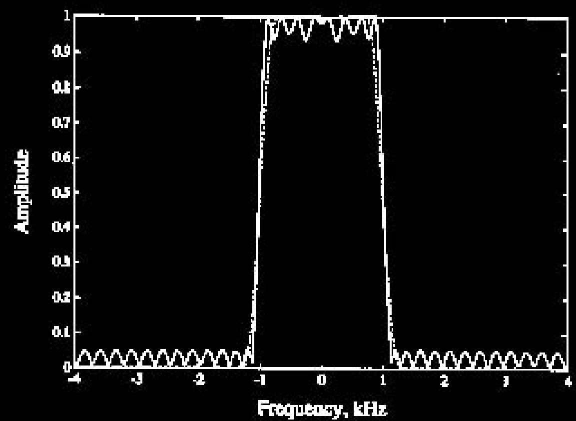
transition width

trade off

RF pulse design

- Tradeoff between ripples and transition width:





--- 0.2% ripple

— 5% ripple

SLR pulse

Slice profile

RF pulse design

2.從B_n獲得A_n:

$$\left|A_n(z)\right|^2+\left|B_n(z)\right|^2=1 \Leftrightarrow \left|A_n(z)\right|=\sqrt{1-\left|B_n(z)\right|^2}$$

--A_n 是固定的，但其實B_n是有很多選擇的

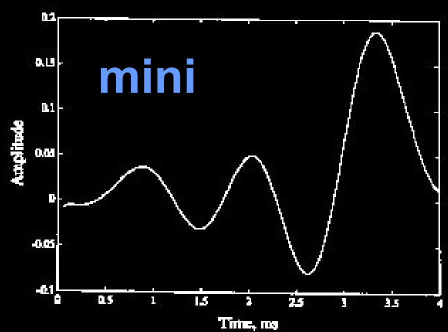
RF pulse design

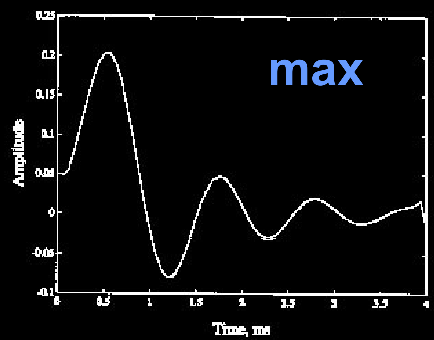
- Different design pulses

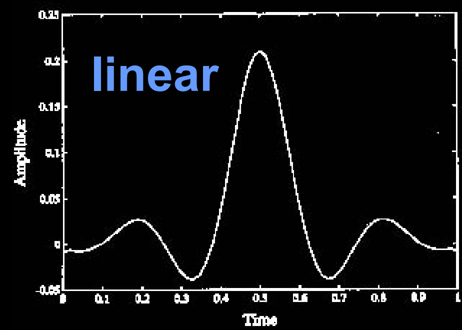
  --minimum phase pulse(when phase is not important)

  --maximum phase pulse (saturation, inversion)

  --linear phase pulse(spin echo pulse)







RF pulse design

3. 有了 B_n, A_n,反推得 $\phi_n, \ \theta_n, \vec{B}_{1,n}$

$$B_n \Rightarrow A_n$$

$$\begin{bmatrix} A_{n-1} \\ B_{n-1} \end{bmatrix} = \begin{bmatrix} C_n & S_n^* \\ -S_n z & C_n z \end{bmatrix} \begin{bmatrix} A_n \\ B_n \end{bmatrix}$$

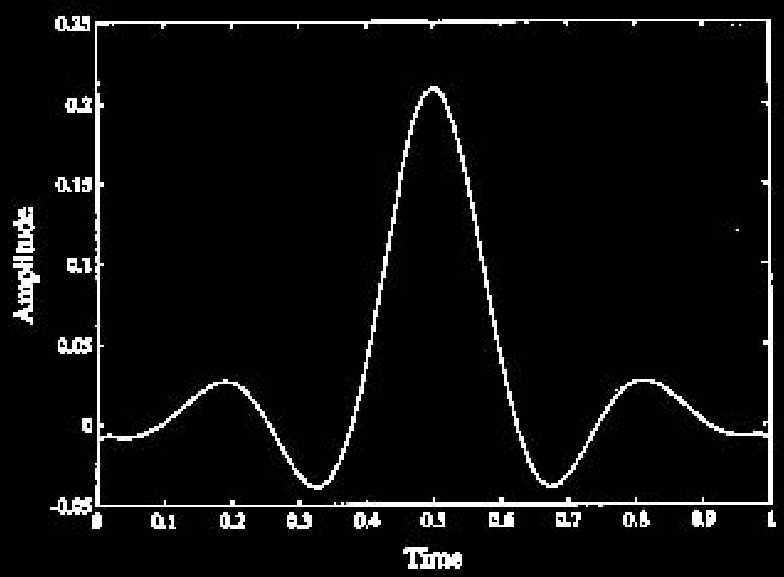
$$\phi_n, \ \theta_n, \vec{B}_{1,n}$$

$$A_{n-1}(z)$$

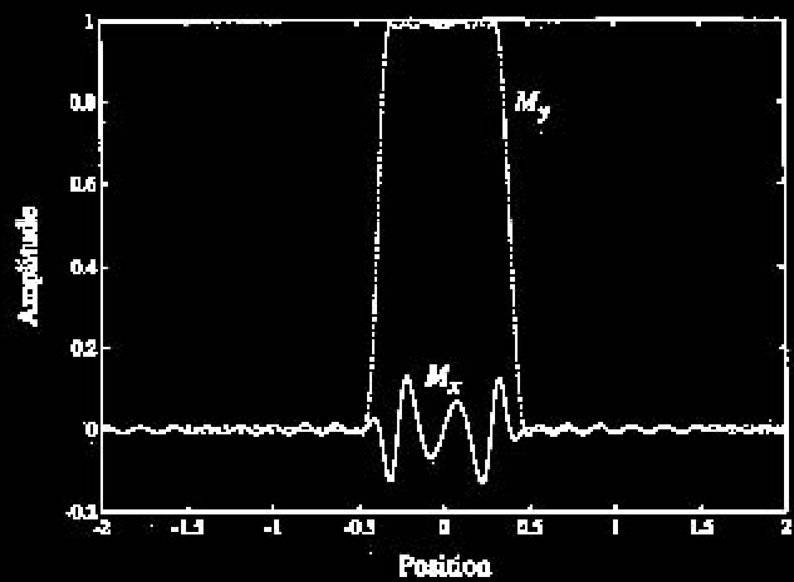
$$B_{n-1}(z)$$

n-1次

RF pulse design



SLR π/2 pulse



Slice profile

Section VI

Application

Application

- Ultra-short TE images

  --for minimum-phase SLR pulses, the isodelay is much shorter

  ➔shorter slice rephasing lobes

  ➔reduced minimum TE

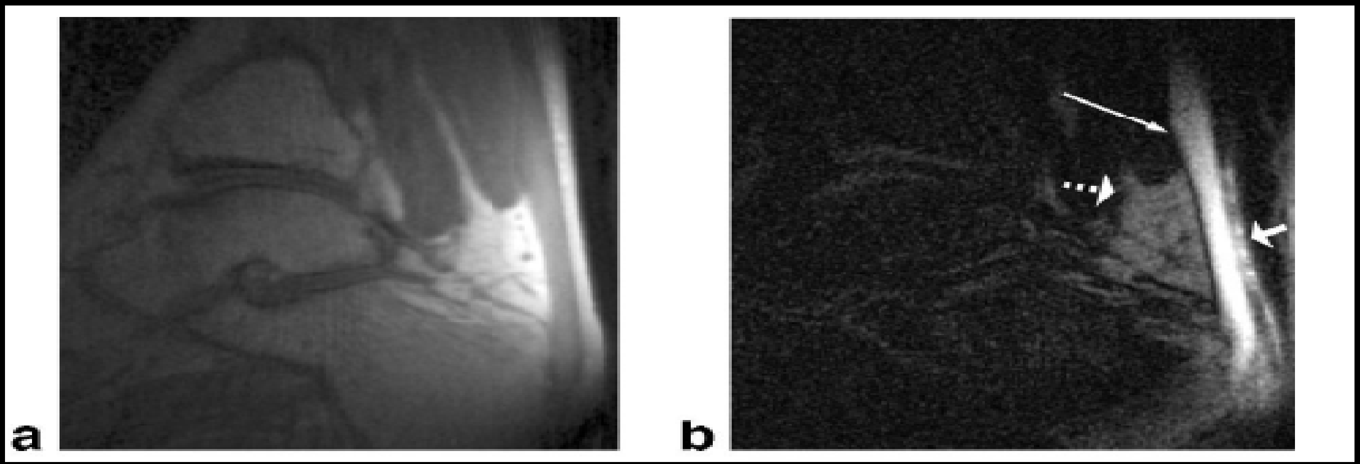
詳細情形, 下星期將為您揭曉…

Application

- 3D UTE ankle image

  --improves the contrast of the tendons

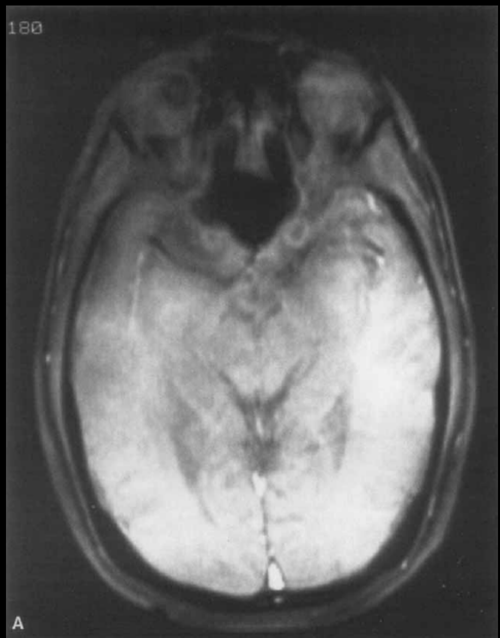
  --TR=100 msec, TE=80 usec



Peder E. Z Larson et al MRM 2006

Fat Suppression?

- 原來上星期也說過



其實更早前老師就講過

特殊射頻脈衝

- ~~Shinnar-LaRoux 射頻脈衝設計法~~
- k-space RF 脈衝設計
- 同一脈衝達到兩種以上目的
- 例：同時做頻率及切面選擇

抱歉，這裡
請大家記
得更正

Basic MRI 第六講 Artifact

Application

- Mercedes **SLR** Stirling Moss

  --3500萬台幣

  --全球限量11部

Conclusion

- SLR transform is suitable for all angle excitations, including the 180°

  --Bloch equation: θ < 1 rad

  --k-space for RF design: θ <90 °

- You can generate what you want

References

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- E.T. Jaynes, *Physical Review*, 1955, 98(4); 1099-1105
- MRI pulse sequence handbook: Ch2~Ch3

Thanks for Your Attention