

1. The excited signal in nuclear magnetic resonance often follows approximately a single exponential decaying waveform multiplied by a frequency “carrier”, called the free induction decay (FID). For an FID signal with envelope in the form of

$$S(t) = \begin{cases} Ae^{-t/T^2}, & 0 \leq t < \infty \\ 0, & t < 0 \end{cases} \quad [1]$$

please analytically calculate its Fourier transform (i.e., its frequency-domain spectrum), and use a computer to plot the real and imaginary parts. This is called the Lorentzian lineshape consisting of an absorption (real) and a dispersion (imaginary) shape.

2. For tissue with T2 equal to 200 msec, use a computer to plot the time domain waveform of its envelope as expressed by Eq.[1] above from $t = 0$ to $t = 100$ msec. Use any software you like (Matlab recommended) to perform a one-dimensional Fourier transform on this signal to plot its spectrum (For Matlab, command “fftshift” is recommended). Do the real and imaginary parts look like the shape derived in the previous question (note particularly whether the dispersion shape gives you smoothly curved maximum and minimum)? Why or why not? What happens when T2 is equal to 2 msec and you still plot from $t = 0$ to $t = 100$ msec? Explain the reasons in sufficient detail such that I know you clearly understand.
3. To give you a rough idea about how tough this course is, try doing the follow computer homework problem even if you have not learned anything from this course yet. The formation of an MR image is through 2D Fourier transform of the two-channel raw data. Hence you can always simulate the raw data of an MRI by performing 2D Fourier transform of an existing image. Now, discuss in theoretical terms what happens if there is an imbalance in the gain of the two receiver channels. Then use computer simulation to show that your inference is correct. Hint: Use any MR image as an example, Fourier transform it to simulate the raw data before image reconstruction. The raw data have two components (real and imaginary) corresponding to the signals received by the two channels. Give them different weightings and Fourier transform back to see the artifacts. Vary the weighting factor and see how the artifacts change in appearance. Then construct your theory to explain the source of artifacts. For those of you who are familiar with Fourier transform properties, consider the odd and even functions in the case of 1D Fourier transform.

4. Calculate the spin population difference (which results in the bulk magnetization) of the two energy levels at body temperature (37°C) when the proton spins ($I=1/2$) are experiencing a magnetic field of strength (a) 1.5 Tesla, and (b) 3 Tesla. Express the above answer in units of ppm (parts-per-million) with respect to the total population. What kind of signal-to-noise ratio (SNR) improvement (due to spin population difference alone) do you expect in your MR images if you perform experiments at 3 Tesla rather than 1.5 Tesla? For Boltzman or Planck constants please refer to textbooks of College Physics. Note that the units of Hz and radians/sec must be taken into account, otherwise your answer will be off by a factor of 2π somewhere.
5. Re-derive, from the very beginning, the Bloch equation for the magnetic moment \vec{M} in the rotating frame by yourself, excluding all relaxation terms. Think carefully about the physical meanings implied in every step.
6. What is the difference in RF power amplifier requirements of an MR system when performing RF excitations for the protons versus the phosphorus nuclei? Answer the above question by calculating the magnetic flux density (i.e., B1 amplitude) in unit of Gauss for a 90-degree RF pulse which lasts for 1 msec in time duration, for both the hydrogen and phosphorus nuclei. Note from basic electric circuit theory that power is proportional to the square of the driving current. Assume for simplicity that the RF pulse is of rectangular shape, and neglect the RF power difference due to difference in frequency (although this is not the case in reality). The gyromagnetic ratio for ^{31}P is 17.25 MHz/Tesla.
7. Given a series of 90° RF pulses spaced TR apart in time (i.e., RF pulses applied at $t = 0, 1*TR, 2*TR, 3*TR, \text{etc.}$), use your computer to plot the longitudinal and transverse magnetization (in unit of fraction of the thermal equilibrium magnetization M_0) as a function of time from $t = -TR$ to $t = 4*TR$. Assume for simplicity that $T_2 \ll TR$ and T_1 is somewhat larger than TR. Also, assume the duration of RF pulses to be very short and negligible.
8. How to measure T1 relaxation time? Explain the meanings of inversion recovery and saturation recovery.
9. Compare Hahn echo (which uses a single 180° pulse) with Carr-Purcell echo (which uses multiple 180° pulse), especially in terms of the effects of molecular

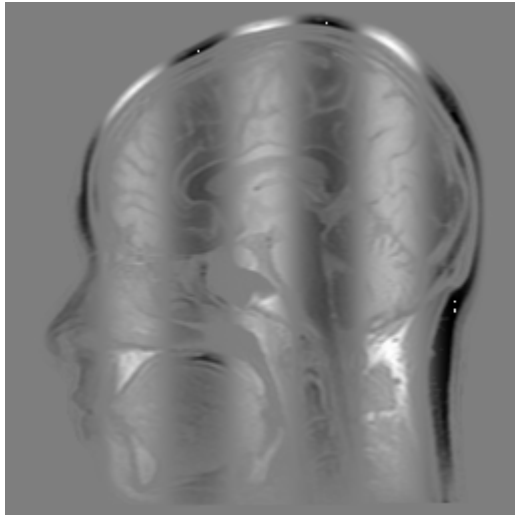
diffusion. When measuring T2 using the above two methods, which T2 measured, do you think, is longer? Compare the difference between the two methods that generate a series of spin echoes, i.e., Carr-Purcell echo and Carr-Purcell-Meiboom-Gill echo, especially in terms of an imperfect calibration of the RF pulse flip angle. Which T2 measured is longer?

10. Design the parameters (TE's) in a 4-echo CPMG RF pulse sequence that helps measuring T2 using curve fitting, in the presence of uncertainty caused by random noise. The four echoes are equally spaced in time. The T2 value is on the order of and 80 msec or so. Give your rationale for selecting these parameters.
11. Calculate the strength of the magnetic field gradient needed to excite a 5mm slice, using a sinc shape RF pulse with one main lobe plus two side lobes at each side, lasting a total of 3 msec. What is the gradient strength needed if you would like to reduce the slice thickness to 2mm? Given the maximum gradient strength of 1.0 G/cm due to hardware limitation before about 1995, what methods can you do to achieve a 2mm slice thickness? Try to think and do not just directly use the formula in the class notes.
12. Use Fourier transform to design the RF pulse waveforms (in the time domain) that give you the following slice profiles in the presence of a slice gradient with maximum amplitude 5 mT/m (= 0.5 G/cm): (a) Two excitation bands, each with 5 cm thickness, spaced 10 mm apart from each other. The null band of 10 mm is located at the isocenter. (b) An excitation band of 10 cm thickness, but with flip angle linearly increased from one end to double at the other end. For both problems, appropriately truncate your RF pulses such that the time durations would not be too long (several msec is ok). Then Fourier-transform back the time-domain waveforms to plot the actual slice profiles. Note that you might need two RF channels in some cases.
13. (a) Calculate the strength of the magnetic field gradient needed for frequency encoding given: field-of-view (FOV) 20cm, sampling frequency 32KHz. What is the pixel width (i.e., spatial resolution) along the frequency encoding direction if 256 complex data are sampled? From the spin-echo pulse sequence diagram, roughly estimate the minimum echo time (i.e., minimum T2 decay possible). (b) Let's say if you want to increase the spatial resolution by reducing FOV to 4cm. What is the gradient strength? Given maximum

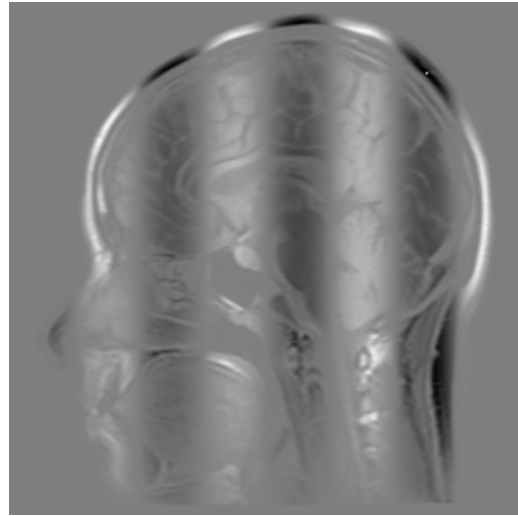
gradient strength of 1.0 G/cm in about 1995, how could you achieve the spatial resolution? What effect does your method have on the minimum echo time?

14. Make sure you understand phase encoding. Calculate the maximum strength and stepping of the phase encoding gradient which lasts for 4 msec, if FOV = 20 cm and a matrix size of 256 phase encodings is used. What should be altered if FOV = 20 cm but matrix size changed to 128 (i.e., resolution lowered by a factor of 2)? What if both FOV and matrix size reduced by half (i.e., FOV = 10 cm, 128 phase encodings, with resolution unchanged)?
15. The gray matter and white matter in human brain have the following relaxation times at 1.5 Tesla: T1(GM) ~ 600 msec, T2(GM) ~ 110 msec, T1(WM) ~ 400 msec, T2(WM) ~ 90 msec. Find the parameter for optimal T1 and T2 contrast, respectively, between gray and white matter when (a) TR=2000 msec with varying TE, and (b) TE=20 msec with varying TR, assuming that the transverse magnetization at the end of TR is small enough to be negligible. Note that since reported T1 and T2 values vary a lot, the optimal parameters need to be roughly estimated only (TR in multiples of 100 and TE in multiples of 10 msec is adequate; Too many digits are meaningless). Assume they have same proton density (although it is not the case in practice). For those who hate mathematics, feel free to use computers to draw a plot and give your estimate qualitatively.
16. What happens to the longitudinal magnetization in finite TR due to T1 relaxation in an 8-echo multi-echo spin-echo sequence ($TE(n) = n \cdot 15$ msec)? Answer the above by using a computer to calculate the longitudinal magnetization as a function of TR for the cerebrospinal fluid (CSF) at TR = 500, 1000, 1500, and 2000 msec. Assume CSF has a T1 of 4500 msec and that T2 relaxation results in no residual signal at the end of TR.
17. The appended images were obtained via separate real/imaginary Fourier transform, respectively. Frequency encoding is along the horizontal direction. Note that gray is zero and white and black are positive and negative maximum values, respectively. Shown also is the desired image (Note that black is zero and white is maximum). Discuss the source of the banding and its solution. From the above why do you think we need magnitude Fourier transform? If it turns out that the phase is also important, how could you refine your pulse sequence (such as the timing for example) quantitatively from the given image?

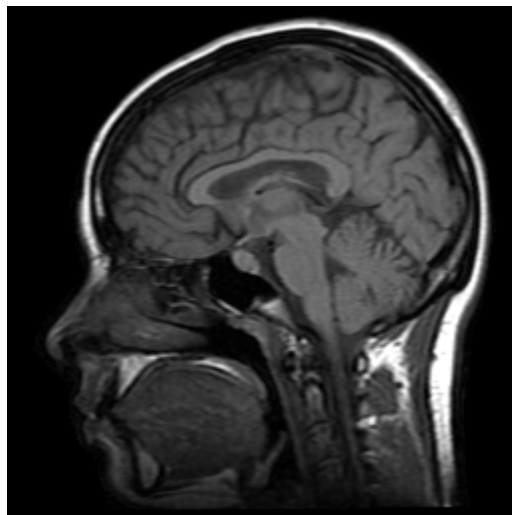
Problem 17



real image



imaginary image

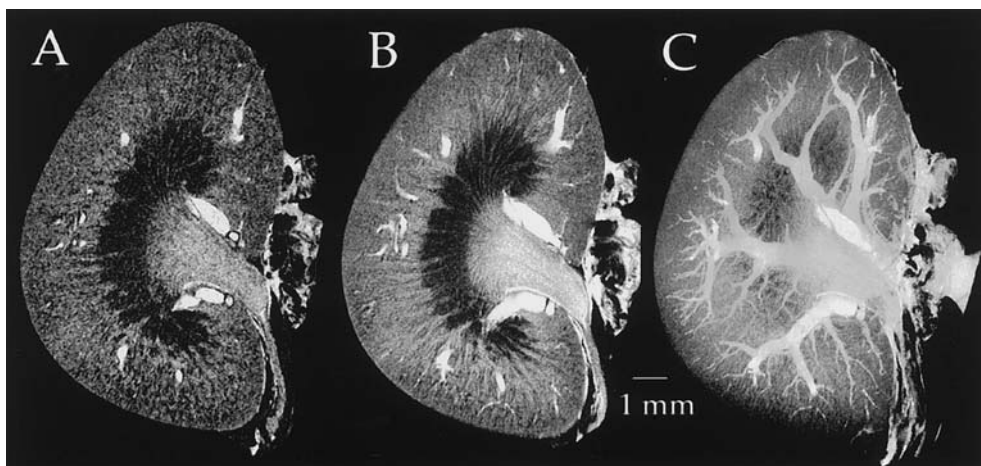


desired image

18. Discuss in theoretical terms what happens if the receiver gain were set too low or too high when sampling and digitizing the MR signals (i.e., the echo). Recall that most MRI receivers have ADCs with only 16 bits. Turning the receiver gain to too low or too high a value may cause the quantization error to be significant or a receiver overflow saturation, respectively. Use a sample image, Fourier transformed to simulate the raw data in the k-space, modify the raw data to simulate the receiver gain effects, and Fourier transform back to yield the image. Compare the image obtained via your simulation to justify your theoretical induction. Note in particular that the effect of

receiver-gain-too-low can be seen only when it is set way too low such that a substantial amount of data get truncated to zero.

19. An MRI magnet has an inductance of 100 Henries (Note: This means a huge inductance.) and operates at a steady current of 50 Amperes. Assume that the field is approximately homogeneous within a cylindrical volume 30 cm in diameter and 60 cm long, and negligible outside of that volume. Calculate the magnetic flux density in Tesla and the stored energy. How long would it take to charge up such a magnet from a power supply operating at a DC voltage of 1 volt? (Hint: $U = 1/2 L I^2 = 1/2 B H V = 1/2 \mu_0 H^2 V$, where U is the stored energy, L the inductance, I the operating current, V the volume, and H the actual magnetic field strength, but in units of A/m. “B” as we used in class to stand for magnetic field strength is actually the magnetic flux density, in units of Gauss or Tesla. Do not use the formula directly. Try to find where they come from.)
20. In an international symposium a speaker mentioned about the potential of MR imaging to achieve 25-micron resolution with a 512 matrix along the readout direction. Spin-echo imaging was used with full-echo $TE = 5\text{msec}$. Estimate the gradient strength in units of mT/m needed for such a purpose. Comment on the requirements on electric current and winding turns of the gradient coil as compared with current clinical state-of-the-art (2008) gradient strength of 40~60mT/m. What is the most convenient way to achieve such a gradient strength requirement for imaging with 25-micron resolution at 512 matrix? Hence, is this kind of high resolution directly applicable to human MR studies in vivo?



Problem #20 Rat kidney images at 25-micron resolution (Radiology 2002)

21. For a circular single-loop RF coil with a radius of 3 inches, calculate the electric current, in units of Ampere, needed to generate a 90° B1 pulse for protons within 1 msec (Note: the parameters used are the same as Problem #3 in your Homework #1). Assume for simplicity that a rectangular waveform is used. Can you use a wire just like those used in electronics laboratory to make the RF coil? Why or why not? For those of you who are not graduates from electrical engineering department, the wires commonly used in electronics laboratory often carry electrical currents on the order of mA.
22. From electromagnetic statics theory, show, mathematically, that if a circular Helmholtz pair is used for receiving coil, the signal at the center of the pair can be received uniformly. You can use the fact that transmitting and receiving are essentially the same thing (reciprocity theorem), i.e., it is sufficient to show that B1 from identical currents in the coil is homogeneous at center of the pair. Note: By “homogeneous”, we mean that the B1 field varies “slowly” with location. The mathematical representation is given by $dB1/dx = 0$ and $d^2B1/dx^2 = 0$ (and all other higher-order differentiation if you can).
23. Is a circular Helmholtz pair, placed at both sides of the object to be imaged, better than a single loop coil (if with same radius) in terms of sensitivity (SNR)? Answer the question above by comparing the sensitivity of the two configurations with the object placed at the centers of them (i.e., assuming a rotating magnetic moment at center of coil and compare the induced Faraday electromotive force). Plot their B1 profiles along the coil axis of circular symmetry. For integrals such as that in the single loop coil, a hemisphere surface may be a good choice. Using the reciprocity theorem is also an option. But if you want to apply the reciprocity theorem, make sure to compare the two configurations at the same driving current applied to the “coil ensemble”, otherwise your answer will be off by a factor of 2.
24. Discuss the effects on the image if the receiver channel has a constant DC offset arising from the electronics. How could you retrospectively correct it? Given are 2 raw data files obtained (256x256 data points each, corresponding to the real and imaginary components. All data points contain 4-byte integers. <http://www.mrilab.org/course>). Could you test your proposed method successfully on it? Discuss how you made it or why you were unsuccessful.

25. Use the same set of data to try implementing half Fourier imaging which saves you about half of the scan time. Make sure to include DC offset correction and phase correction in your reconstruction program. Does it work well? Why or why not? Is the SNR measured same as your theoretical prediction? Note that almost all the commercial MRI systems in the world have the half Fourier imaging function in their standard package. So there must be some way to make it work.
26. Given that $T2^*$ in human tissues are on the order of 40 msec at 1.5 Tesla, you would like to finish a scan within that time using EPI. Neglecting the time occupied by the blipped phase encoding gradients (why can you ignore them?). What is the sampling frequency needed if 128×128 data points are to be sampled (compare with the commonly used sampling frequency of 32KHz)? What is the gradient strength needed for a head scan, say, $FOV = 24$ cm (compare with the maximum gradient strength of 1.0 G/cm on a 1994 GE Signa MRI)? Why can't we lessen the gradient hardware by reducing the sampling frequency? Qualitatively discuss what would happen if the blipped gradients occupy some time? Qualitatively discuss what would happen if the gradient rise time is not negligible? Then discuss why EPI cannot be used on older MRI systems. Given that $T2^*$ roughly scales in inverse proportion to the main field strength, comment on the use of EPI on 3.0 Tesla or 7.0 Tesla MRI systems.
27. Use any one transaxial MRI image of the brain, simulate the presence of Nyquist ghost (or $N/2$ ghost) in EPI due to inconsistency in neighboring k-space lines resulting from the back-and-forth trajectory of EPI (phase direction is anterior-posterior). The simplest simulation, although also the farthest from what happens in real world, is probably amplitude inconsistency where you need only k-space manipulation. Closer to the practical situations are timing or phase inconsistencies where you need the imaging equation. But since the latter is not easy for those unfamiliar with programming, the simplest simulation is acceptable. State your processes clearly in sufficient details.
28. Predict the spacing of the ghost artifacts in terms of breathing frequency and TR, assuming a simple periodic motion. Design an experiment to really see the motion ghost located as predicted (i.e., give me a set of numbers). Optionally at your spare time, try it on any MR system you can access and verify your prediction.

29. Calculate the spatial shift in units of pixels, along both the frequency and phase encoding directions, versus bandwidth (i.e., sampling frequency, usually 8K, 16K, 32K, or 64K) at 1.5 Tesla due to chemical shift difference between water and fat protons (~ 3.5 ppm). In what portions of the human body do you think is this kind of experiment most suitable? Or what kind of phantom can be used? Design your experiment to really see the chemical shift effects (i.e., choose an anatomic location and a readout bandwidth). Optionally at your spare time, try it on any MR system you can access and verify your prediction.
30. For the CHESS pulse sequence using bipolar RF pulses, compute the frequency profile for 1-1, 1-2-1, 1-3-3-1, and 1-4-6-4-1 configuration. Which one gives sharpest selection? Determine the time interval for the entire RF pulse duration as the time between the starting and the ending RF pulses at 1.5 and 3.0 Tesla.
31. For a typical single-shot EPI for proton, estimate the maximal allowable readout reference frequency offset in units of ppm (parts per million) that would cause a spatial displacement by less than half a pixel at 3.0 Tesla field strength. Use typical imaging parameters such as: total scan time ~ 100 msec, 128×128 matrix (therefore what is a rough estimate of the sampling frequency along the frequency and phase encoding directions?), 24 cm FOV, 7 mm slice thickness, and so forth. Neglect the back-and-forth k-space trajectory of EPI. You can also neglect the durations of the blipped phase encoding gradients (Do you know why?).
32. Plot the FID signal (i.e., transverse magnetization) as a function of time (from $t = 0$ to $t = 20TR$) when a series of RF pulses spaced TR apart are applied with flip angle of (a) 60° and (b) 10° . Roughly how many RF pulses are needed to drive the signal into steady state? Assume for simplicity that at the end of the TR , the transverse magnetization can be destroyed to zero (i.e., spoiled). Do the above for two tissues with $T_1 = 15TR$ and $T_1 = 3TR$. Let $T_2^* = TR/4$.
33. Calculate signal strength versus TR and flip angle for SPGR sequence (What is SPGR? Look into your reference by General Electric and see what fast pulse sequence mentioned in class is just like SPGR. Siemens calls it FLASH. Marconi calls it RF-FAST. Philips calls it FFE. Also notice that for imaging you essentially apply a train of RF pulses. The signal reaches a steady state

after several RF pulses (see the problem above) so that you don't even need to calculate the transient-state signal). What flip angle gives you optimal SNR? What kind of contrast is it at such a flip angle? Discuss how to achieve PD and T1 weighting using SPGR sequence without changing TR.

34. You know that short-T2 objects will be blurred along the phase encoding direction in FSE images using low-high phase encoding order. What happens if FSE is used to image an object which contains two substances with distinct T2 values adjacent to each other? Also assume a low-high phase encoding order (i.e., proton density weighted).
35. Compare the spatial shift in units of pixels along the phase encoding directions in single-shot echo-planar imaging (EPI) due to chemical shift difference between water and fat protons (~ 3.5 ppm) at 1.5 Tesla with versus without using phased array RF coil for parallel imaging acceleration. Assume that the reduction factor is 2, which yields a single-shot EPI with about half of the total scan time. The scanning parameters are otherwise constant (i.e., same FOV, same spatial resolution, etc.). Neglect the small difference in the durations for the phase encoding gradients. Is there an improvement or deterioration for the chemical shift artifacts using parallel imaging in EPI?
36. Given a FLASH-like gradient-echo imaging slice thickness of 5 mm, plot the flow-related enhancement as a function of flow velocity from 0, 5, to 10 cm/sec at a TR of 50 msec, assuming bulk flow at constant velocity. T1 of blood can roughly be assumed to be the same as the background static tissue at about 500 msec. Assume equal proton density (although this is not true obviously). Do the above for two flip angles 30° and 90° and compare their results. Note that in FLASH sequence, the residual transverse magnetization gets destroyed at the end of TR, hence neither T2 nor T2* plays a role here. Special note: Your senior classmates' answers to this problem before 2004 were all wrong! Hence do not consult those homework sets.
37. Explain the meaning of flow compensation ("Flow Comp" in GE terms; "MAST (Motion Artifact Suppression Technique)" in Picker terms; "GMR (Gradient Moment Rephasing)" or "GMN (Gradient Moment Nulling)" in Siemens terms) in your own language. Location of protons can in general be divided into many terms including position, velocity, acceleration, and higher order terms. In class we have mentioned the technique to refocus

velocity-induced phase shifts by using gradient moment nulling. Try to derive a gradient waveform for both slice-selection and frequency-encoding gradients that compensates the acceleration effects. Comment on the effect of TE and possible advantages/disadvantages.

38. Design the time length and waveform of a bipolar gradient pair with 2.5 G/cm (25 mT/m) maximum gradient strength to encode flow velocity within ± 20 cm/sec (suitable for encoding CSF flow in human). Draw its waveform and mark out its gradient amplitude and time duration at appropriate locations.
39. Mathematically generate a laminar flow profile in a circular vessel with radius of 10 pixels in a square FOV with width of 30 pixels. Assume maximum velocity of ± 50 cm/sec at a Venc of 30 cm/sec. Simulate the phase image as would be seen in a phase-contrast MR study. This illustrates the phenomena of phase aliasing in phase-contrast MR imaging.
40. Assume that the diffusion-weighted gradients are of rectangular shape and that the time separation between the two gradients can be neglected. On an MRI system with maximum gradient strength of 2.5 G/cm, estimate the minimum echo time in a spin-echo diffusion-weighted imaging sequence with b factor of 1000 sec/mm^2 . Then explain why diffusion-weighted images also show strong T2 contrast (This is called the T2 shine-through effect).
41. From your reading assignments or any references you like, find the exact formula to compute the b factor for diffusion weighting under arbitrary gradient waveforms between RF excitation and signal receiving. Based on this formula, compare qualitatively the effects of diffusion weighting of: (a) a spin-echo imaging sequence with TE = 80 msec (the prephasing gradient is placed before the 180 refocusing pulse); and (b) the fourth-echo image of a multiple spin-echo imaging sequence with TE = 20, 40, 60, and 80 msec. To simplify matters, you can consider only the frequency-encoding gradients, and assume that they are identical in duration and amplitude between (a) and (b). From your inference, tell me whether the fast spin-echo or the conventional spin-echo sequence is more sensitive to diffusion-related signal loss.
42. Among the perfusion-weighted MRI techniques using contrast injection mentioned in class, which one(s), do you think, is (are) suitable for measuring brain perfusion in normal mice at about 3.0 Tesla field strength? Why? Give

me strong reasons in less than one A4 double-spaced page to convince me that you are not simply “guessing”.

43. A graduate student performed a rat fMRI experiment using BOLD contrast at 4.7 Tesla field strength. The box-car design for the stimulus paradigm involved a continuous ON-OFF scheme where the ON represented a breath-in of 5% CO₂ mixed with 95% O₂ (This is called the “carbogen”), and the OFF represented a breath-in of normal air. After a series of experiments the student observed a 20% signal change between ON and OFF in the cerebral cortex. Hence the student concluded that the physiological brain adaptation in the presence of CO₂ resulted in a compensatory increase in cerebral blood flow from vasodilatation that caused the signal increase. Please comment on this study in less than one A4 double-spaced page as if you were the student’s thesis committee member.
44. Knowing that the Nyquist ghost is likely present in essentially all EPI images, comment on the findings of any frontal lobe activations if the fMRI experiments were performed using EPI. Propose possible remedies with the least changes in EPI scanning parameters such as number of slices or TE. Note that you have done the simulation of Nyquist ghost in the past. I am giving this question to you starting 2008 because the Nyquist ghost problem persists in essentially “all” the fMRI protocols used by the Psychology Department at National Taiwan University but has never been seriously discussed by any of these scientists!
45. To give you a rough idea about how tough the MR research is, I’m giving you this homework problem related to something that I have not taught you in this semester, but which happens in routine clinical MR examinations almost every day. The purpose is to let you know that you have NOT learned much in MR even after successful survival from this semester. In proton MRS, the lactate shows a doublet located at 1.3 ppm and separated by about 7 Hz. The 7-Hz splitting is due to the quantum physical phenomena called scalar coupling (or J-coupling) and is independent of the static magnetic field strength. In PRESS spectroscopy, what is the appearance of the lactate doublet at TE = 1/J ~ 135 or 144 msec (i.e., coupling constant J = 7 Hz)? Why? I want a detailed explanation to show me that you understand the concept, using any reference or textbook you can find. The answer should be given in less than five double-spaced A4 pages. Then note: This is not even research yet!