

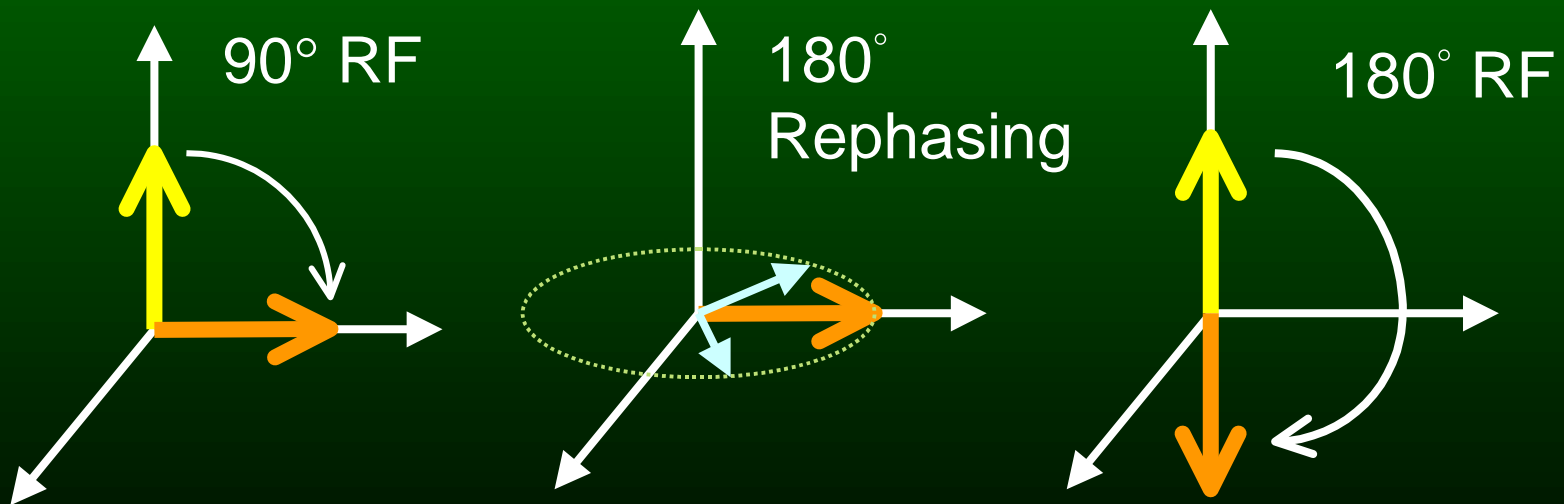
RF Pulse Design

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RF Pulse的目的

⇒ 讓特定範圍內每個spin都能旋轉特定角度



開始設計

⇒ 根據Larmor's Eq.

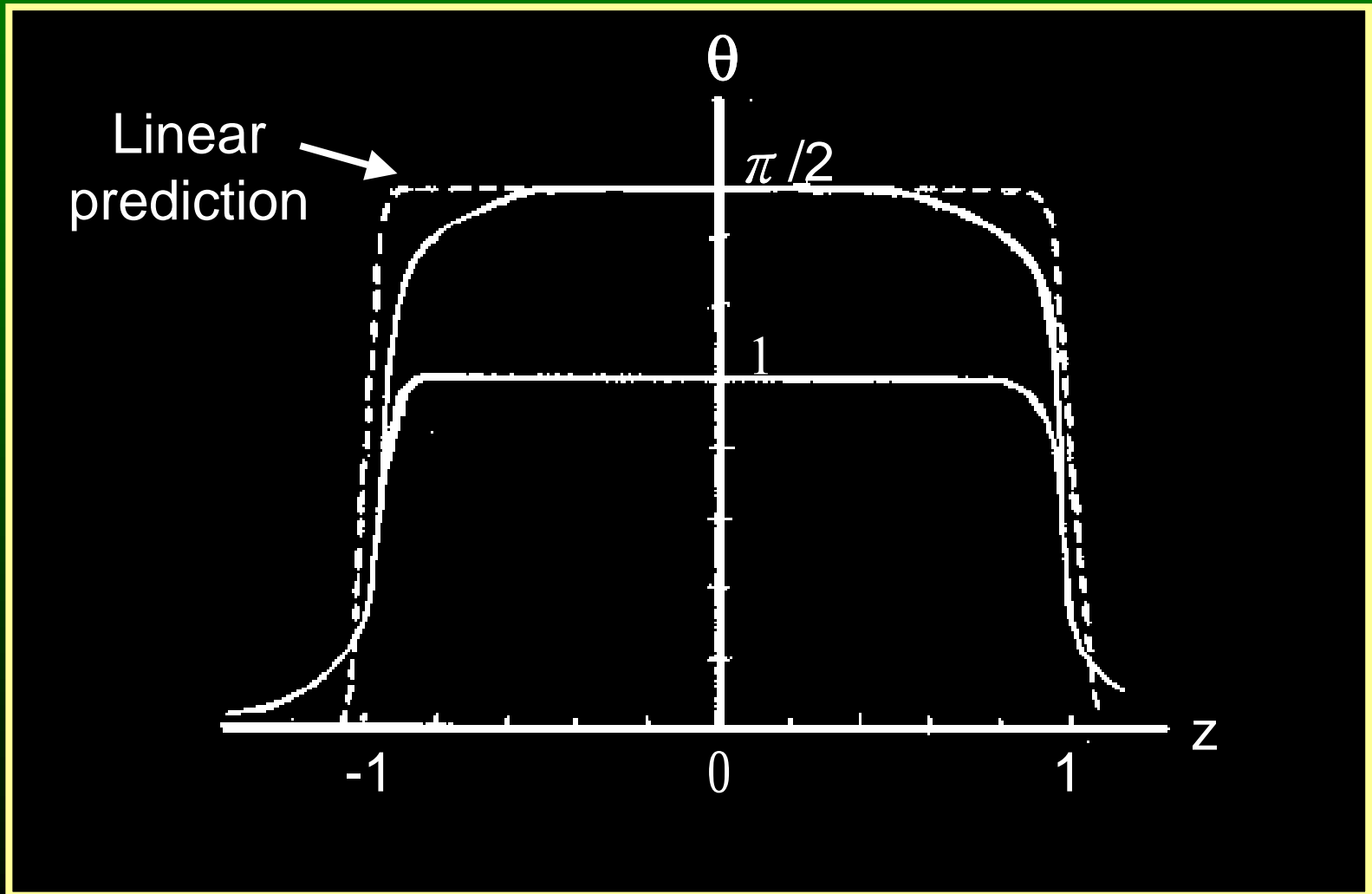
$$\omega = \gamma B \quad \therefore \theta = \gamma B \Delta t$$

⇒ 也就是說，

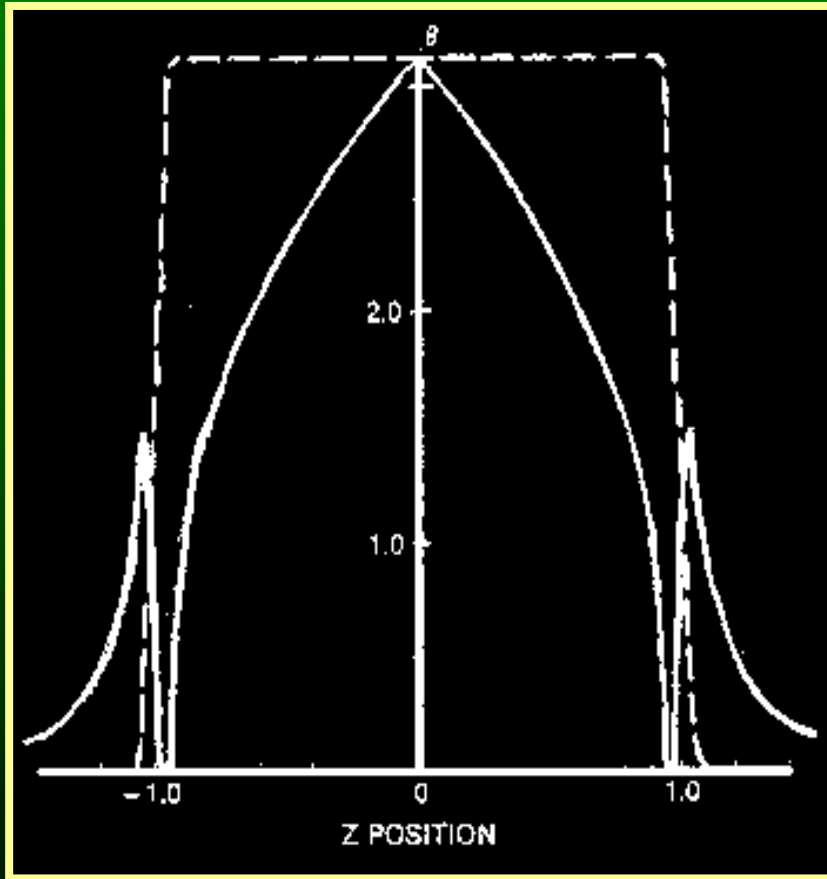
If $B_1 \rightarrow 90^\circ$ RF

then $2B_1 \rightarrow 180^\circ$ RF

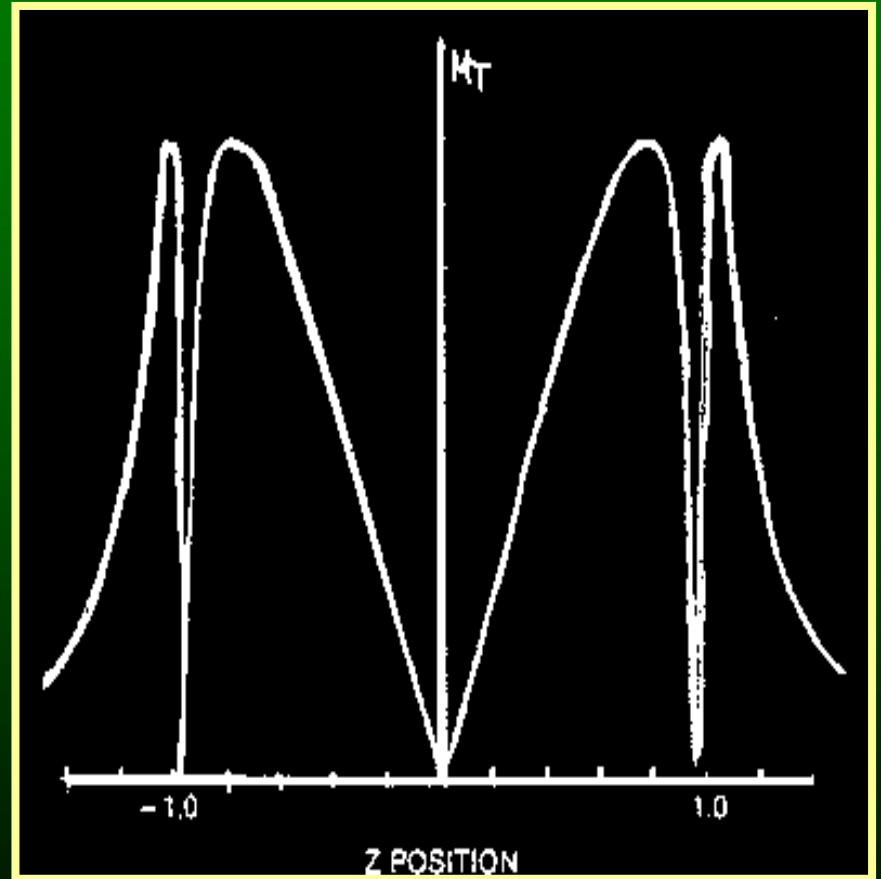
看看結果...



180° RF的結果

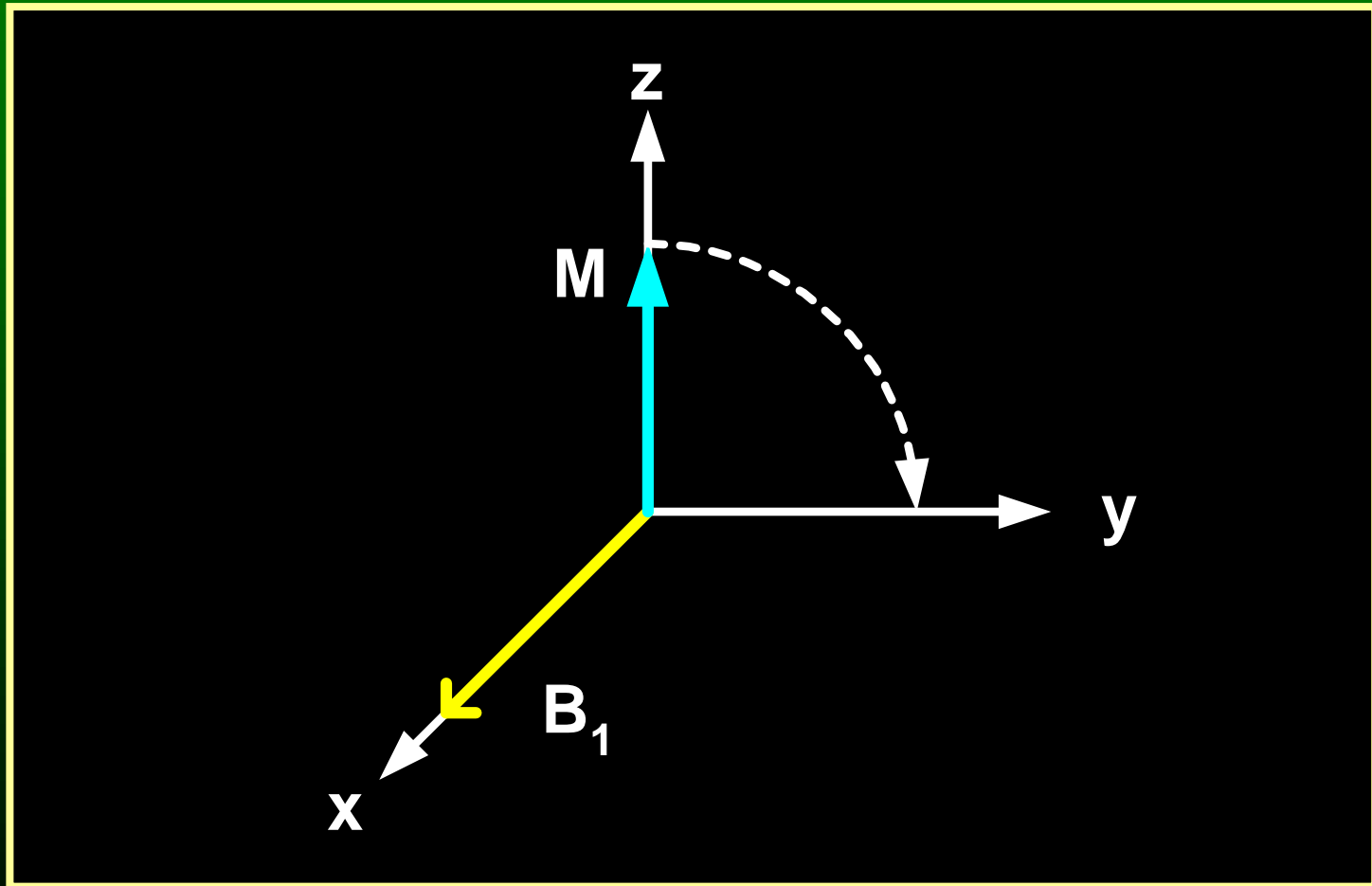


Flip angle v.s. z position

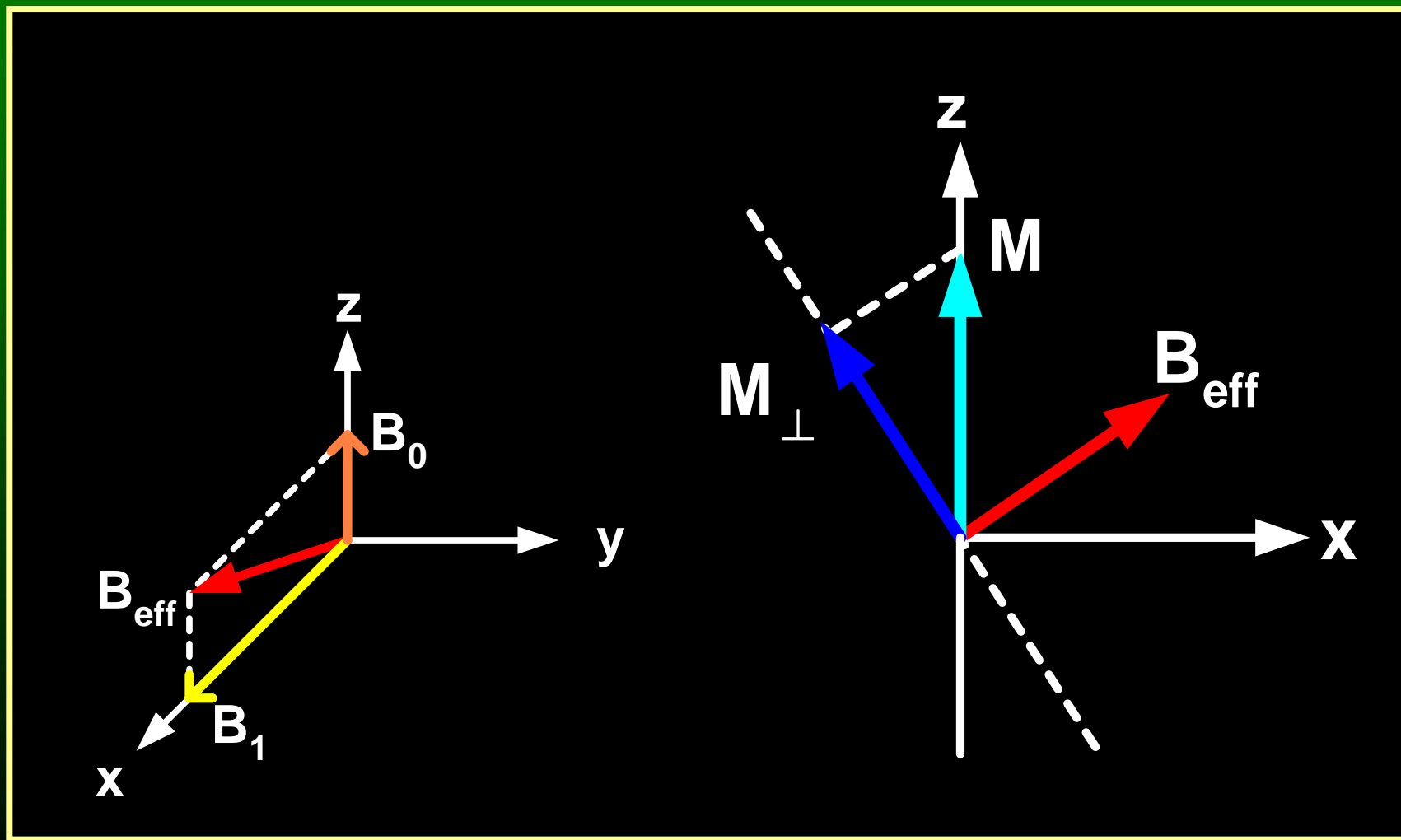


M_T v.s. z position

圖解



圖解



Θ 與 B_1 的關係

$\Rightarrow \theta \propto B_1$ *only when*

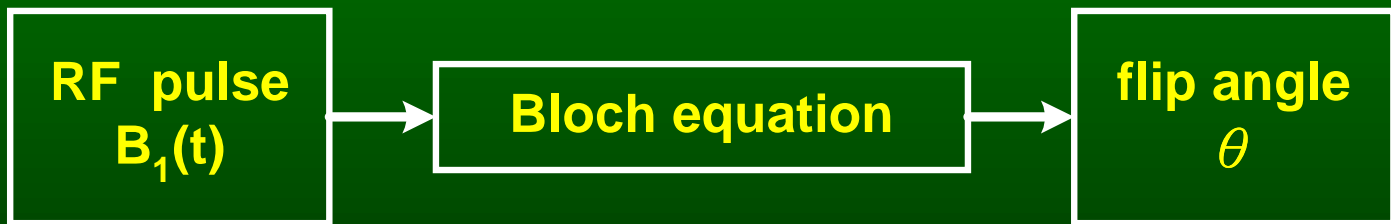
\Rightarrow frequency of B_1 is close to ω_0

\Rightarrow off-resonance

\Rightarrow nonlinear between θ and B_1

$\Rightarrow \theta < 180^\circ$

前因後果



Bloch Equation

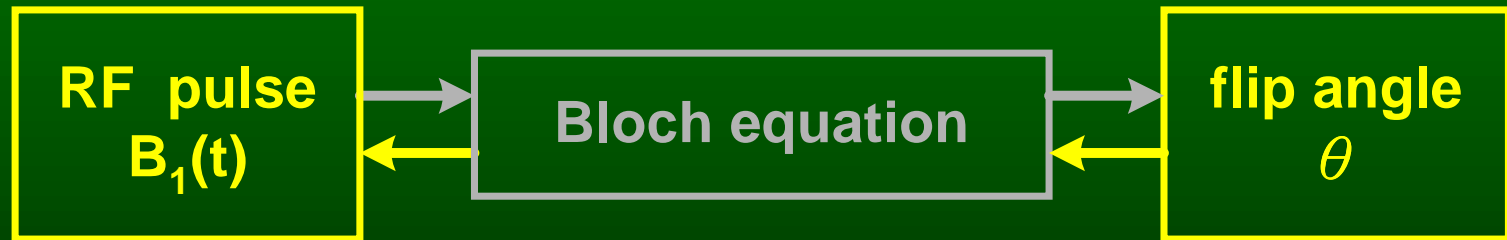
$$\frac{d\vec{M}}{dt} = \gamma \vec{M} \times \vec{B}_{eff}$$

- ⇒ M 跟 B_1 根本不會成正比！
- ⇒ 解微分方程，才能找出 θ 與 B_1 的關係

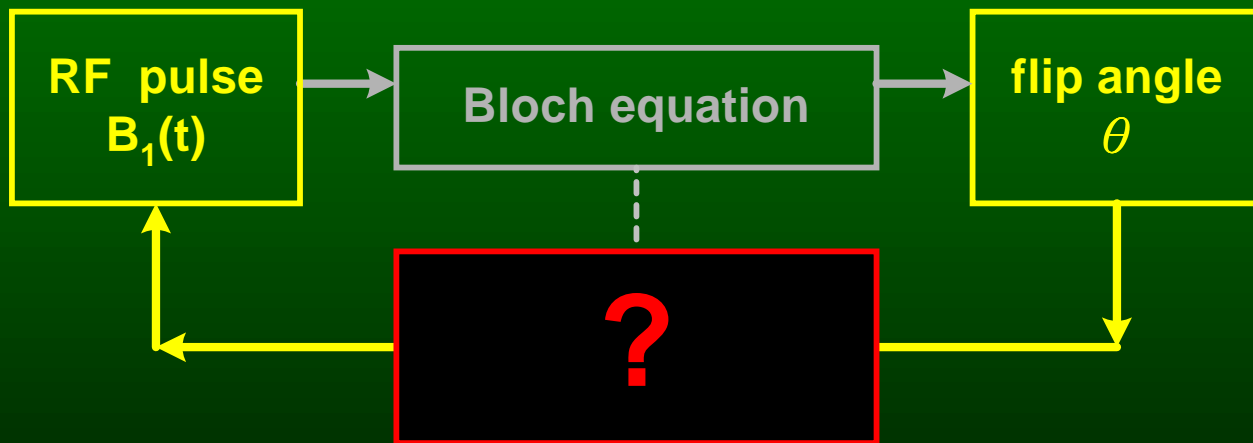
問題是...

What we want
is
what we need

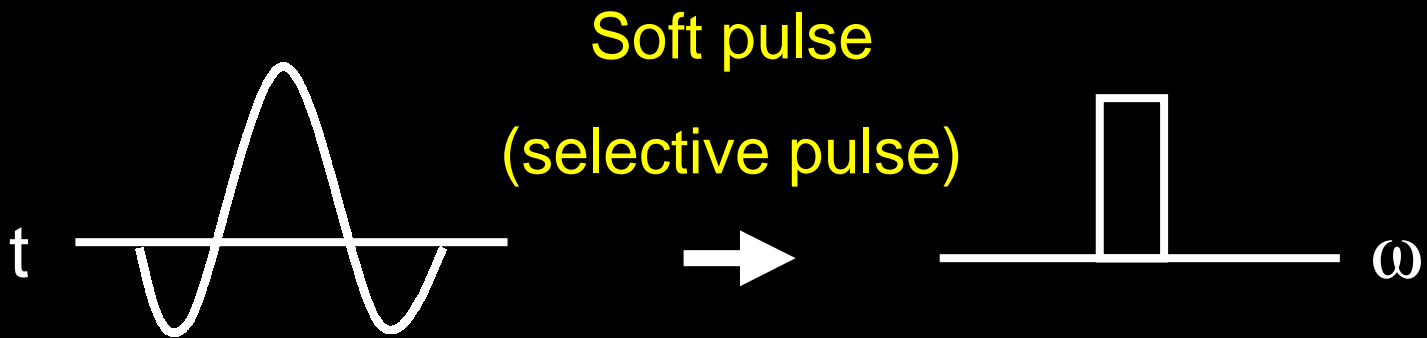
What we want ...



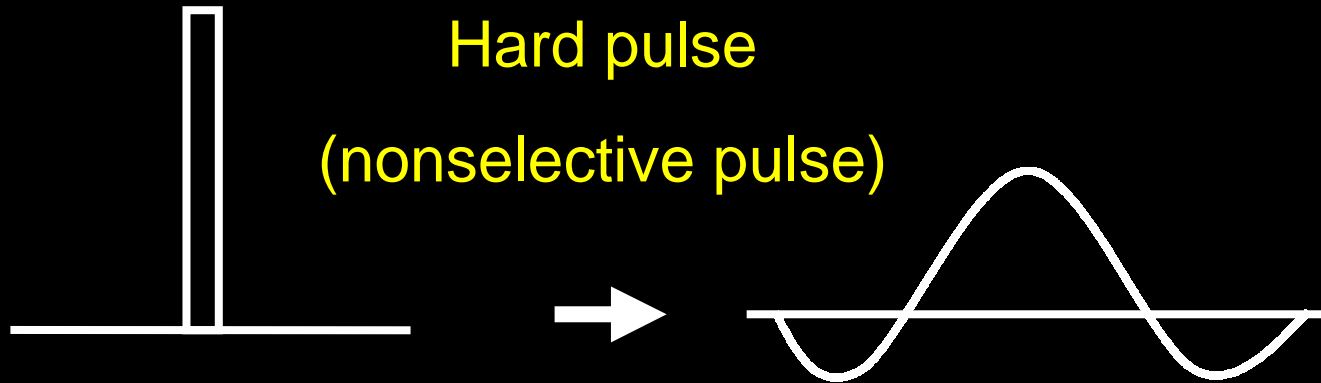
What we need...



Soft & Hard Pulse



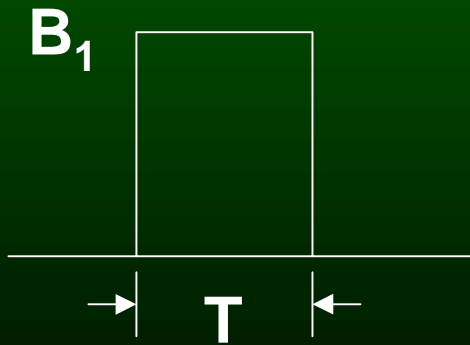
Hard pulse
(nonselective pulse)



針對 $B_1 = \text{constant}$

$$\theta = 2 \sin^{-1} \left[\frac{\gamma B_1}{R} \sin(RT/2) \right]$$

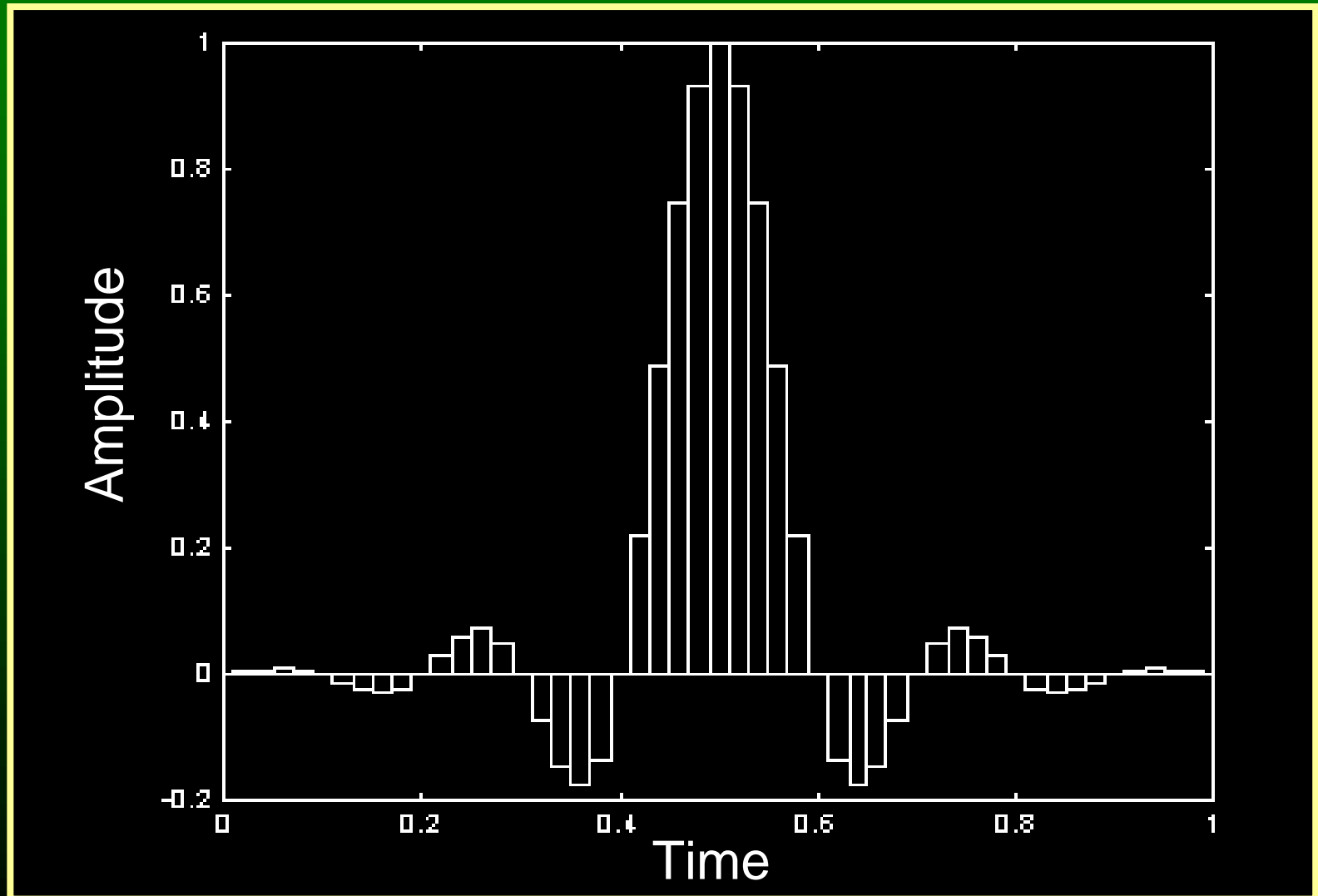
$$R = \sqrt{\gamma^2 B_1^2 + (\Delta\omega)^2}$$



T : B_1 作用的時間

$\Delta\omega$: frequency offset

Hard-pulse approximation

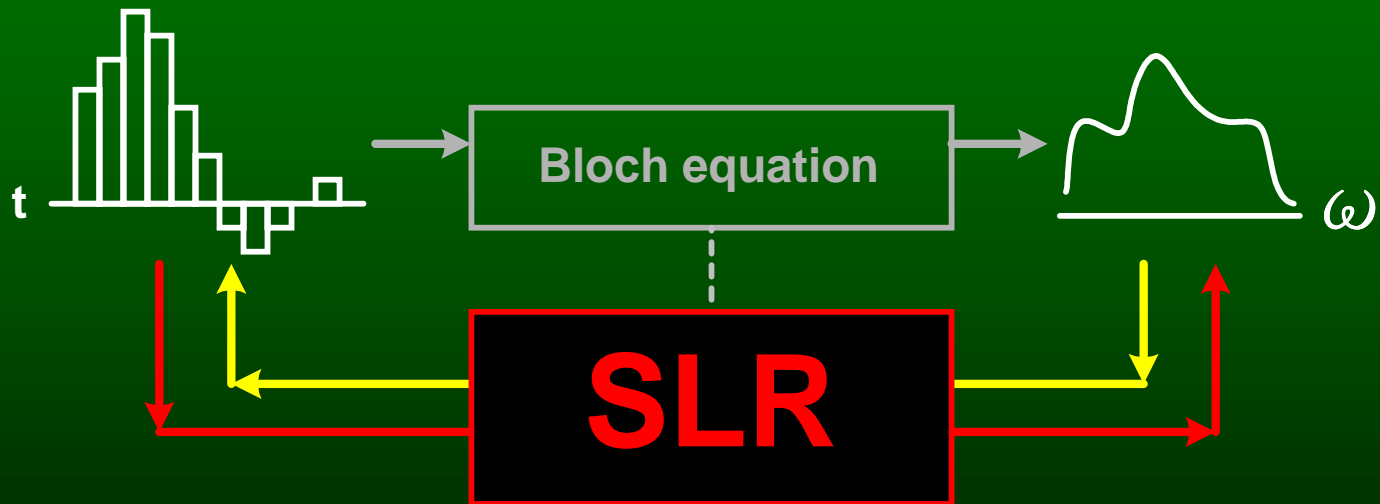


也就是說...

⇒ Hard-pulse sequence design

⇒ 任意波形 RF 皆可產生

Shinnar-Le Roux (SLR)



Bloch eq. 又來了...

$$\begin{pmatrix} M_x' \\ M_y' \\ M_z' \end{pmatrix} = \gamma \begin{pmatrix} 0 & Gx & -B_{1,y} \\ -Gx & 0 & B_{1,x} \\ B_{1,y} & -B_{1,x} & 0 \end{pmatrix} \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix}$$

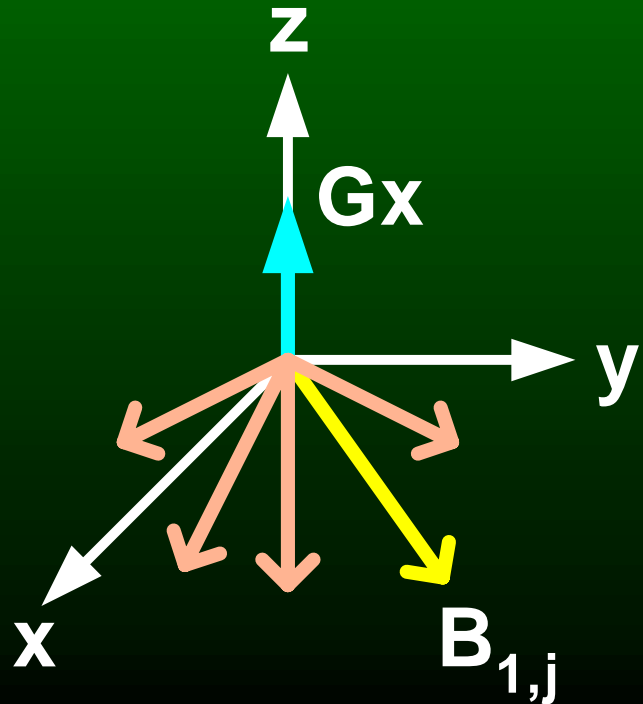
$$\vec{B} = (B_{1,x}, B_{1,y}, Gx)$$

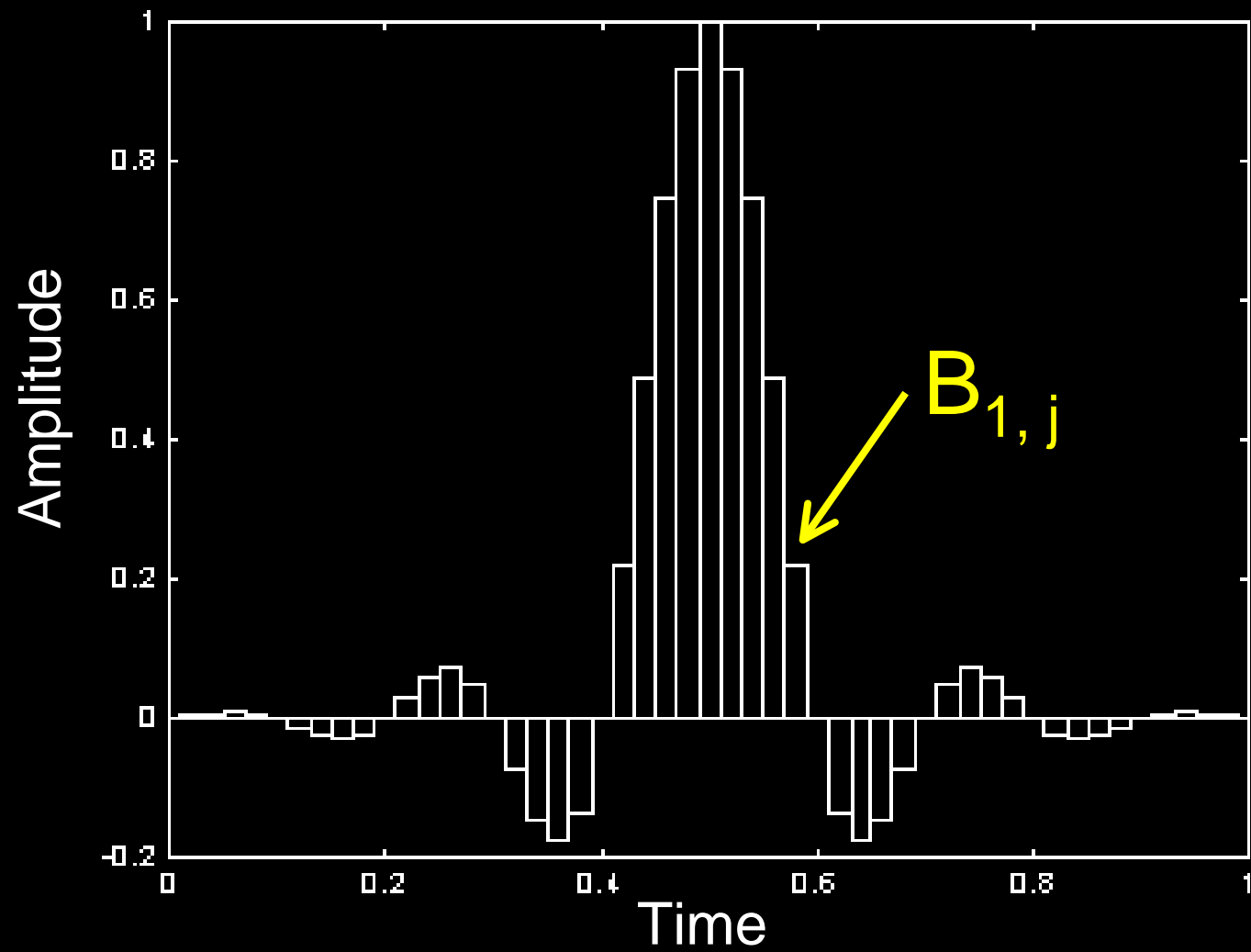
矩陣解法

$$\Phi_j = -\gamma \Delta t \sqrt{|\mathbf{B}_{1,j}|^2 + (\mathbf{Gx})^2}$$

$$\vec{\mathbf{n}}_j = \frac{\vec{\mathbf{B}}}{|\vec{\mathbf{B}}|}$$

$$\mathbf{Q} = \mathbf{Q}_n \mathbf{Q}_{n-1} \cdots \mathbf{Q}_1$$





把矩陣變成多項式

$$C_j = \cos(\gamma |B_{1,j}| \Delta t / 2)$$

$$\begin{pmatrix} A_1 \\ B_1 \end{pmatrix} = \begin{pmatrix} C_1 \\ S_1 \end{pmatrix}$$

$$S_j = i e^{i \angle B_{1,j}} \sin(\gamma |B_{1,j}| \Delta t / 2)$$

$$\begin{pmatrix} A_2 \\ B_2 \end{pmatrix} = \begin{pmatrix} C_2 C_1 - S_2^* S_1 z^{-1} \\ S_2 C_1 + C_2 S_1 z^{-1} \end{pmatrix}$$

$$Z = e^{i \gamma G_x \Delta t}$$

$$A_n(z) = \sum_{j=0}^{n-1} a_j z^{-j}$$

$$B_n(z) = \sum_{j=0}^{n-1} b_j z^{-j}$$

做個整理吧...

$$B_1(t) \stackrel{\text{SLR}}{\Leftrightarrow} (A_n(z), B_n(z))$$

$$\Rightarrow A_n(z), B_n(z)$$

$$\Rightarrow A_{n-1}(z), B_{n-1}(z), \dots, A_1(z), B_1(z)$$

$$\Rightarrow B_1(t)$$

問題又來了...

⇒ 那... $A_n(z)$, $B_n(z)$ 怎麼算??

⇒ $B_n(z)$ -- 逼近 slice profile

- ▶ FIR digital filter

- ▶ Parks-McCellan (PM) algorithm

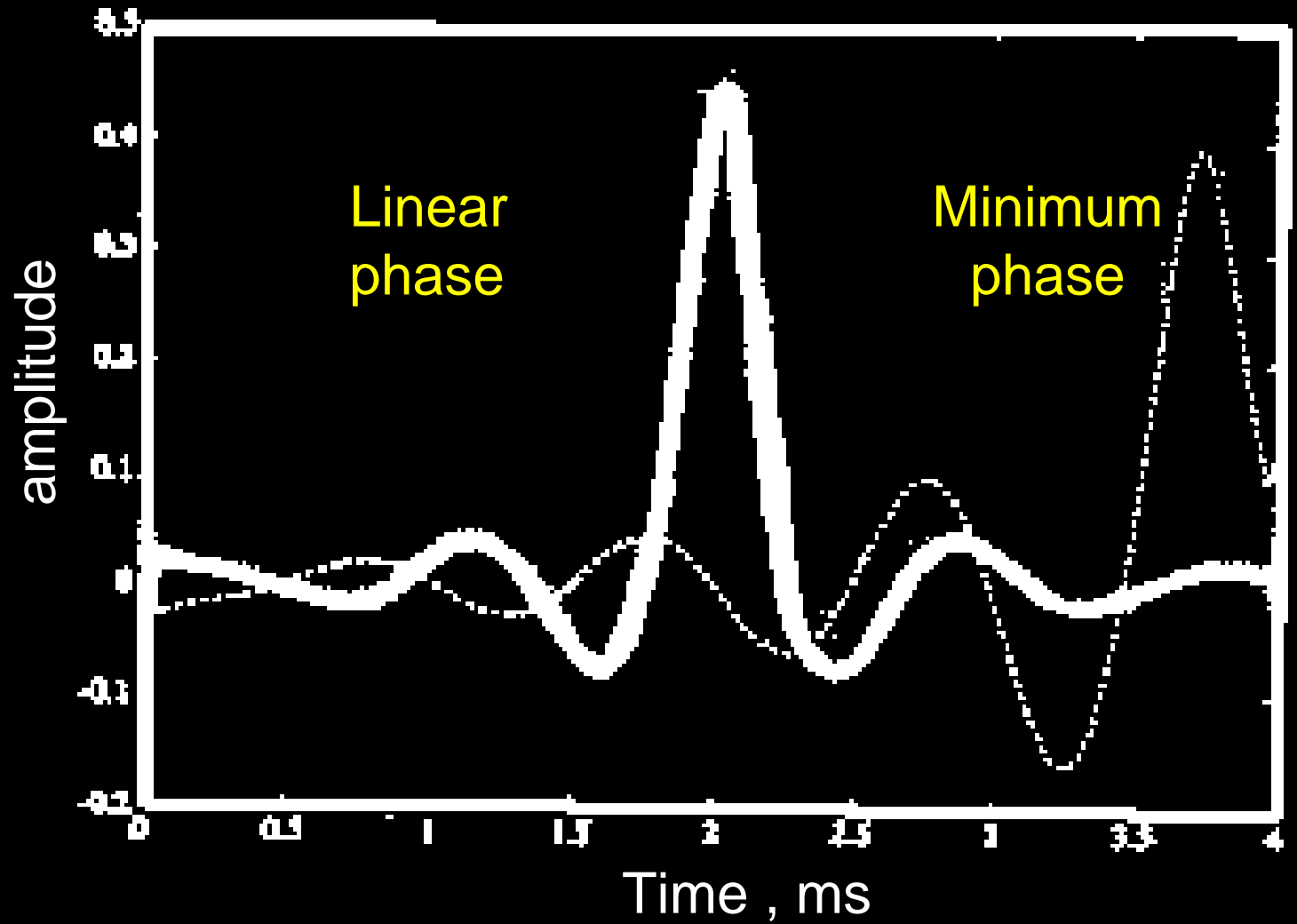
- ▶ ripple, transition bandwidth, pulse length, slice width, RF power

$B_n(z)$ to $A_n(z)$

$$\Rightarrow |A_n(z)|^2 + |B_n(z)|^2 = 1$$

$$\Rightarrow A_n(z)$$

- ▶ linear phase
- ▶ minimum phase
- ▶ maximum phase



做個結論

- ⇒ RF 問題知多少？
- ⇒ Hard pulse approximation
- ⇒ SLR algorithm

