# K-space analysis for RF pulse design

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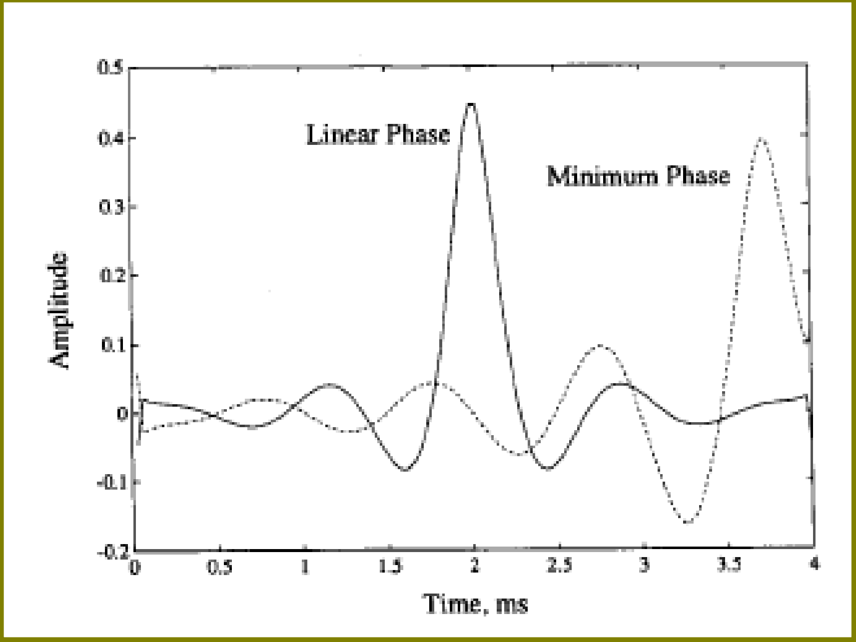
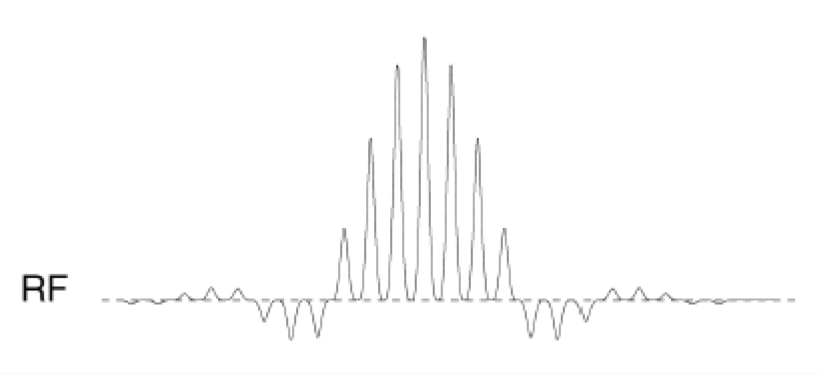
April 15, 2003

# K-space analysis for RF pulse design

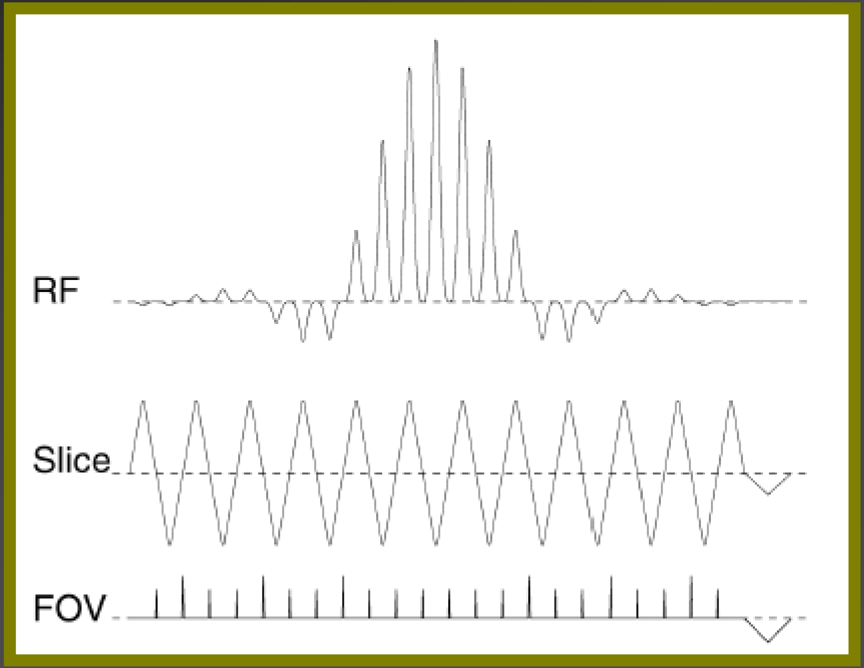
- 上禮拜不是講過了嗎?



# K-space analysis for RF pulse design

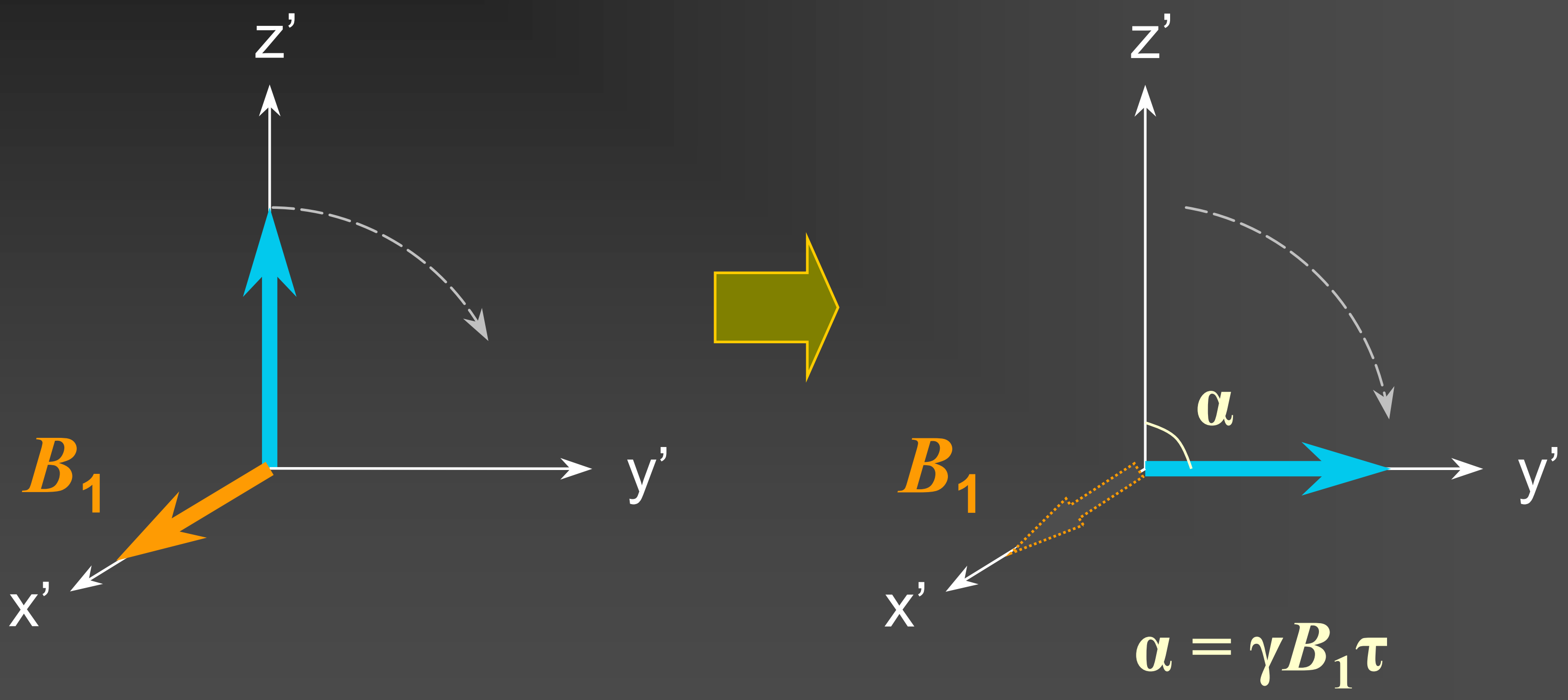
- 除了變化B1, 連gradient都在變



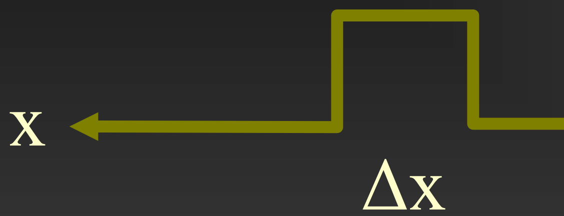
# 大致上分成兩類

- 2-D spatially-selective excitation
- Spatial and spectral selective excitation

# Review: RF Pulse



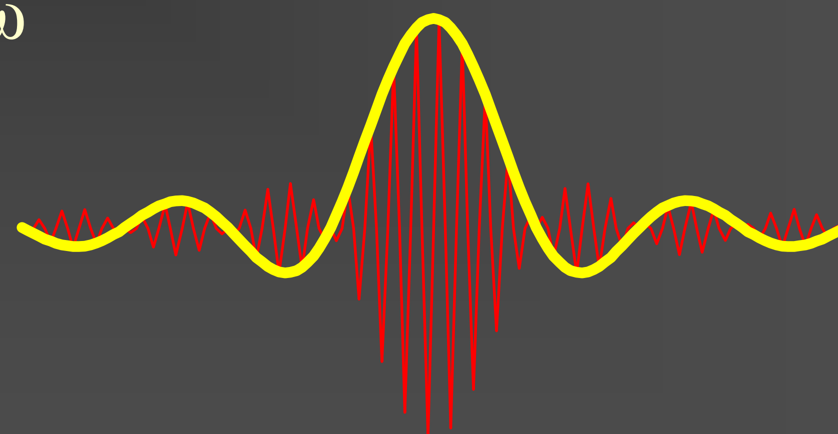
# Review: Slice Selection



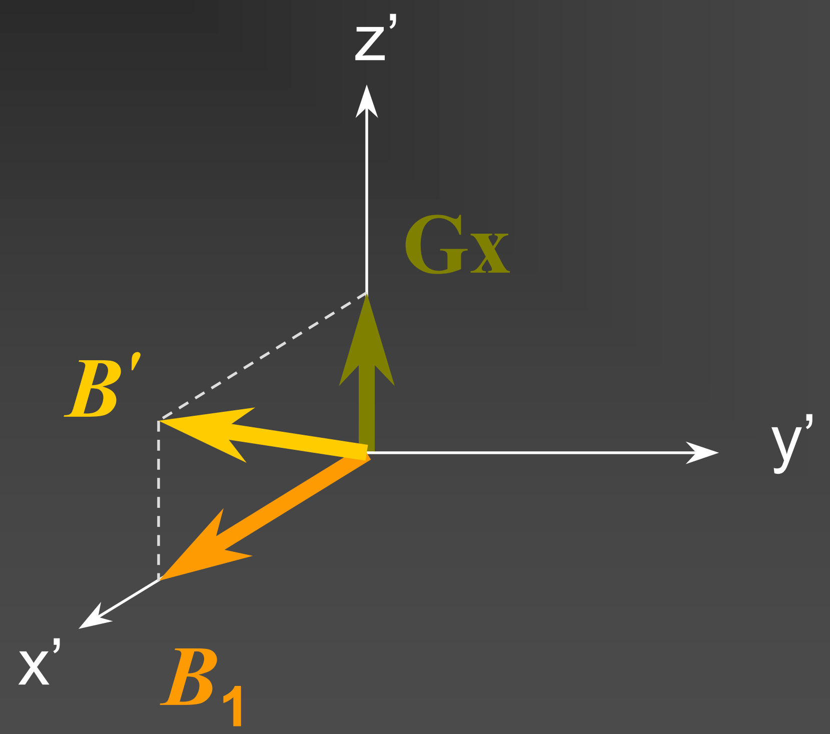

1. Choose a Δx slice.


2. Correspond to Δω.

3. $B_1$ profile



# Review: In Practical,.....



# Bloch Equation

$$\left(\frac{\partial \vec{\mathrm{M}}}{\partial \mathrm{t}}\right)_{\mathrm{rot}} = \gamma \vec{\mathrm{M}} \times \vec{B}' \quad , \quad \vec{B}' = \left(B_{1,x}, B_{1,y}, \mathrm{Gx}\right)$$

$$\begin{pmatrix} \dot{M}_x \\ \dot{M}_y \\ \dot{M}_z \end{pmatrix}_{rot} = \gamma \begin{pmatrix} 0 & Gx & -B_{1,y} \\ -Gx & 0 & B_{1,x} \\ B_{1,y} & -B_{1,x} & 0 \end{pmatrix} \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix}_{rot}$$

# 前提

- Small-tip-angle
- $M_z \approx M_0$ = constant
- Neglecting relaxation
- Non-linear Bloch Equation

# 簡化equation

- Transverse term
  - $M_{xy} = M_x + iM_y$
  - $B_1 = B_{1,x} + iB_{1,y}$

$$\begin{pmatrix} \dot{M}_x \\ \dot{M}_y \\ \dot{M}_z \end{pmatrix}_{rot} = \gamma \begin{pmatrix} 0 & Gx & -B_{1,y} \\ -Gx & 0 & B_{1,x} \\ B_{1,y} & -B_{1,x} & 0 \end{pmatrix} \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix}_{rot}$$

- $\dot{M}_{xy} = -i\gamma \mathbf{G} \cdot \mathbf{x} M_{xy} + i\gamma B_1 M_0$

# 解微分方程

- $M_{xy}(\mathbf{x}) = i\gamma M_0 \int_0^T B_1(t) e^{-i\gamma \mathbf{x}\cdot\int_t^T \mathbf{G}(s)ds} dt$

  $= i\gamma M_0 \int_0^T B_1(t) e^{i\mathbf{x}\cdot\mathbf{k}(t)} dt$

- $\mathbf{k}(t) = -\gamma \int_t^T \mathbf{G}(s)ds$
  - Path through spatial frequency domain
  - **k**(T) = 0

# 大家忍耐一下

- $M_{xy}(\mathbf{x}) = i\gamma M_0 \int_0^T B_1(t) e^{i\mathbf{x}\cdot\mathbf{k}(t)} dt$

$$= i\gamma M_0 \int_0^T B_1(t) \int_{\mathbf{K}} {}^3\delta(\mathbf{k}(t)-\mathbf{k}) e^{i\mathbf{x}\cdot\mathbf{k}} d\mathbf{k}dt$$

$$= i\gamma M_0 \int_{\mathbf{K}} \frac{B_1(t)}{|\gamma \mathbf{G}(t)|} \int_o^T \left\{ {}^3\delta(\mathbf{k}(t)-\mathbf{k}) \left|\dot{\mathbf{k}}(t)\right| \right\} dt \, e^{i\mathbf{x}\cdot\mathbf{k}} d\mathbf{k}$$

W(**k**(t)) — S(**k**)

# 終於…

- $$M_{xy}(\mathbf{x}) = i\gamma M_0 \int_{\mathbf{K}} W(\mathbf{k}) S(\mathbf{k}) e^{i\mathbf{x}\cdot\mathbf{k}} d\mathbf{k}$$
$$= i\gamma M_0 \int_{\mathbf{K}} p(\mathbf{k}) e^{i\mathbf{x}\cdot\mathbf{k}} d\mathbf{k}$$
- $$W(\mathbf{k}(t)) = \frac{B_1(t)}{\left|\gamma \mathbf{G}(t)\right|}$$
- $$S(\mathbf{k}) = \int_0^T \left\{ {}^3\delta(\mathbf{k}(t) - \mathbf{k}) \left|\dot{\mathbf{k}}(t)\right| \right\} dt$$

# 整理一下

- $M_{xy}(\mathbf{x}) \overset{f}{\Leftrightarrow} p(\mathbf{k}) = W(\mathbf{k})S(\mathbf{k})$

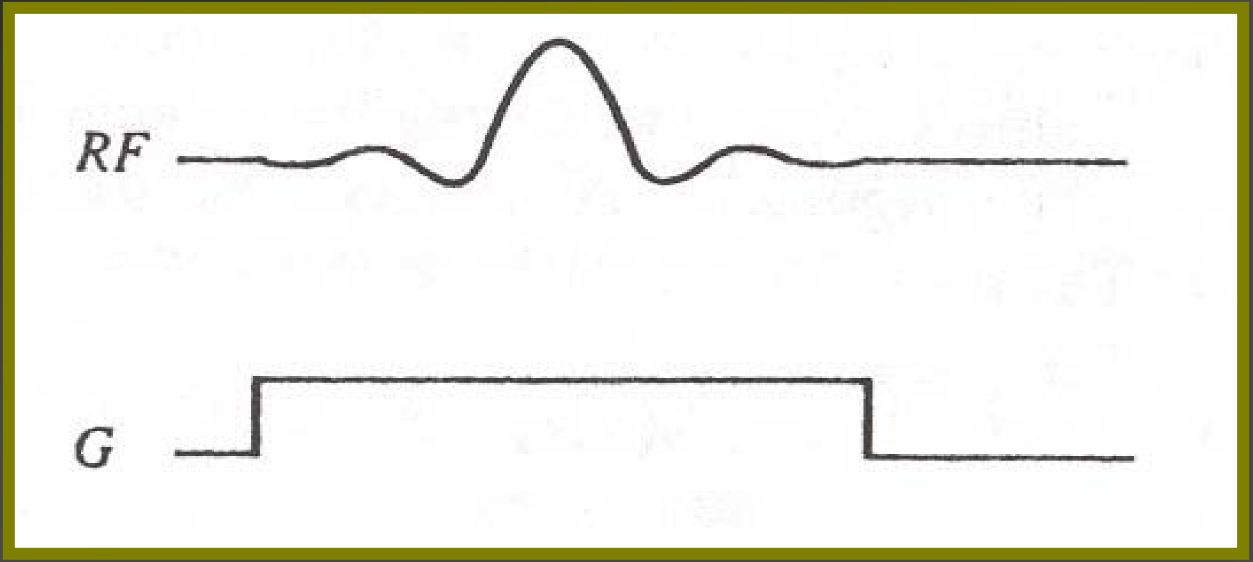
Transverse magnetization

Weighted trajectory

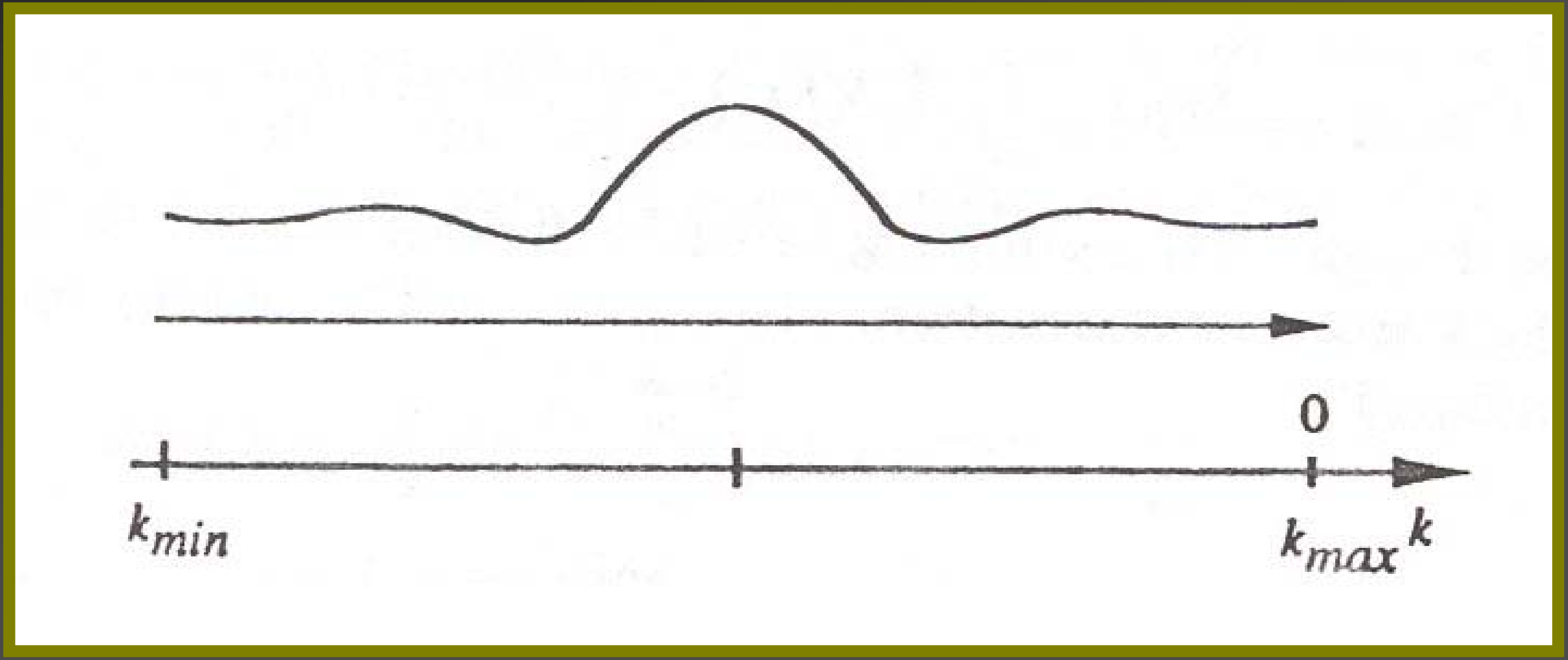
- $W(\mathbf{k})$ : spatial weighting function
- $S(\mathbf{k})$ : unit weight trajectory

# Conventional excitation

- Without focused gradient

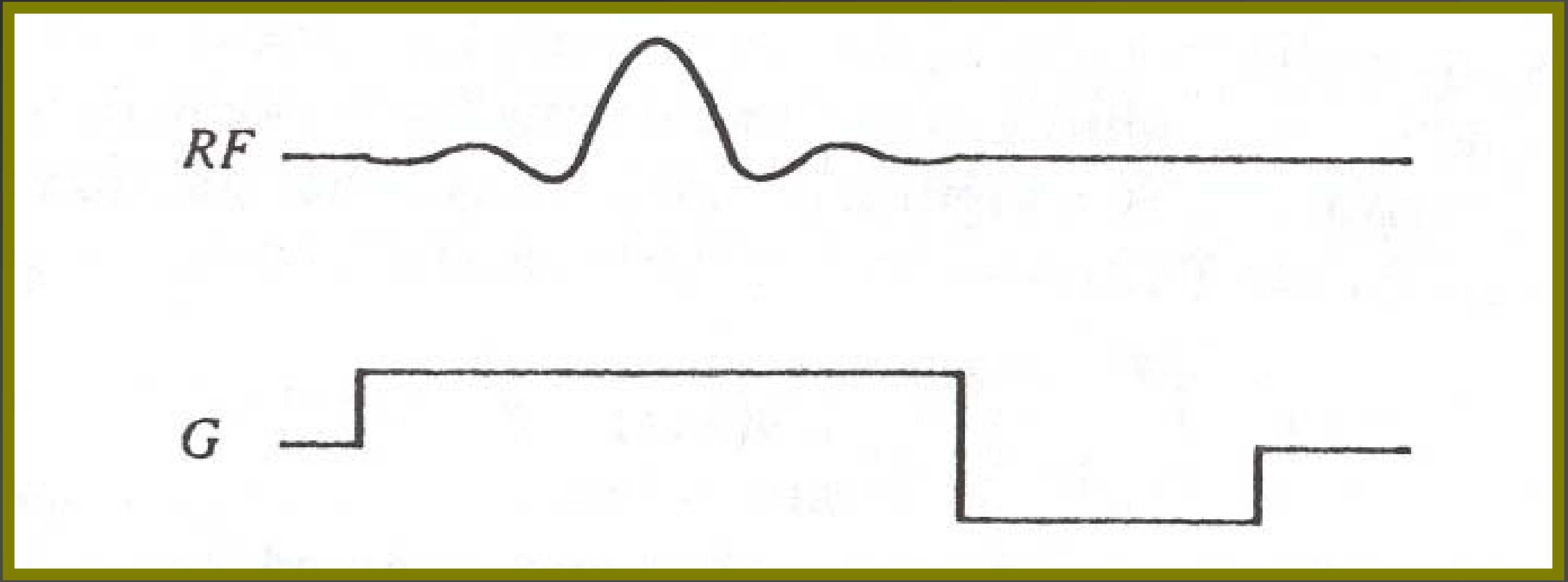


# Conventional excitation

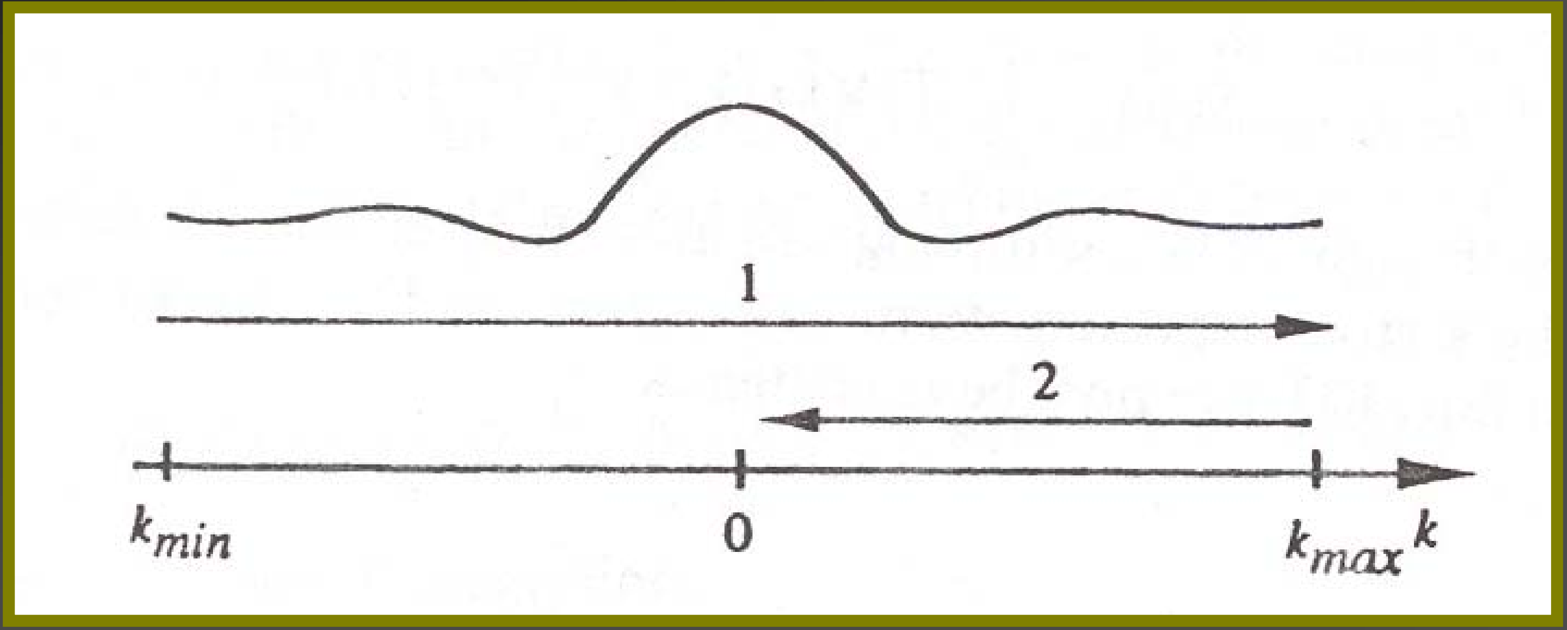


# Conventional excitation

- With refocused gradient



# Conventional excitation



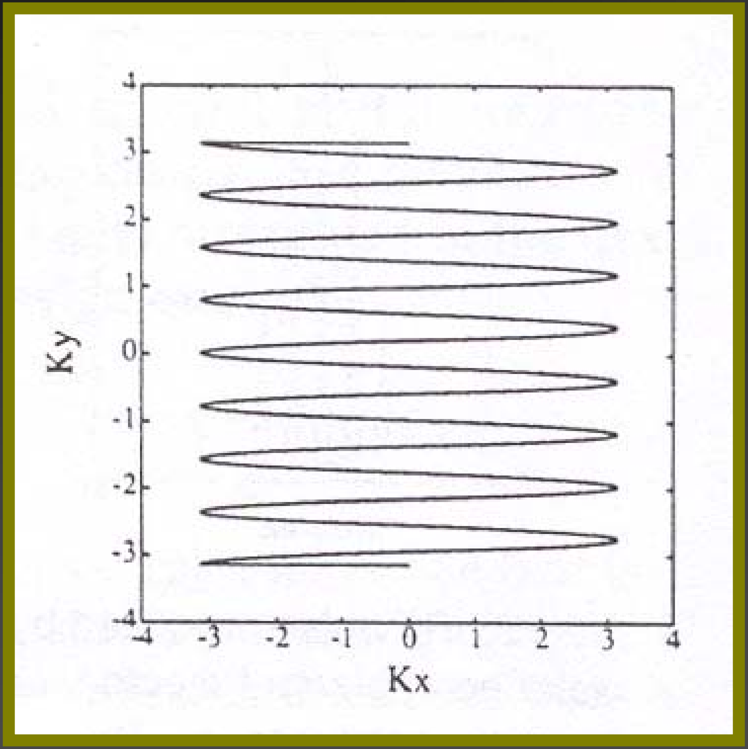
# 2-D selective excitation

- 想要激發的範圍?強度分布?
- Fourier transform → D(**k**)
- Select W(**k**), S(**k**)
- W(**k**)*S(**k**) ~ D(**k**)
- S(**k**) → **G**(t)
- W(**k**) & **G**(t) → $B_1$(**t**)

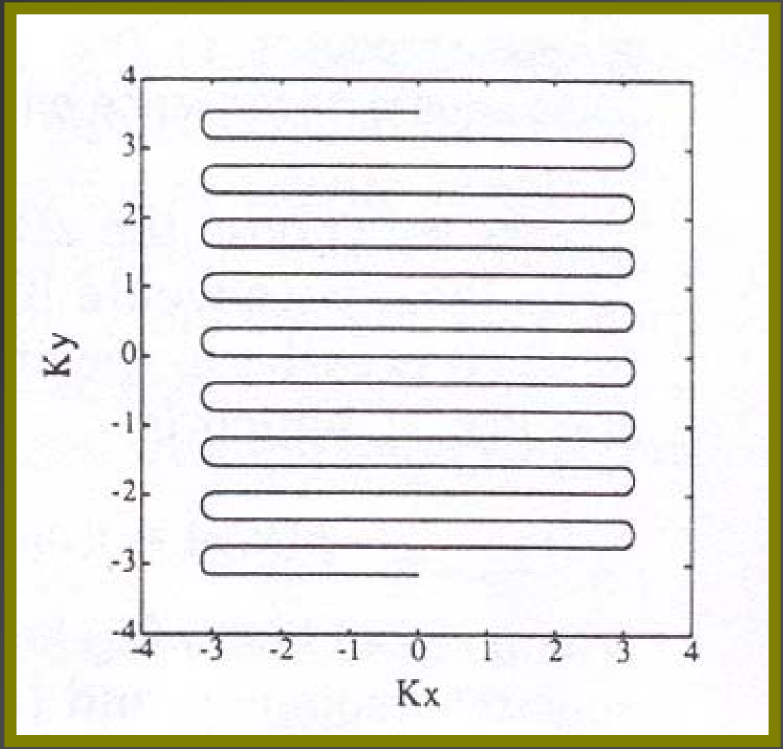
# K-space trajectory

- $S(\mathbf{k}) = \int_0^T \left\{ {}^3\delta(\mathbf{k}(t) - \mathbf{k}) \left| \dot{\mathbf{k}}(t) \right| \right\} dt$
- 跟快速造影有點像
- Echo planar家族
- Spiral 家族

# Echo planar trajectory

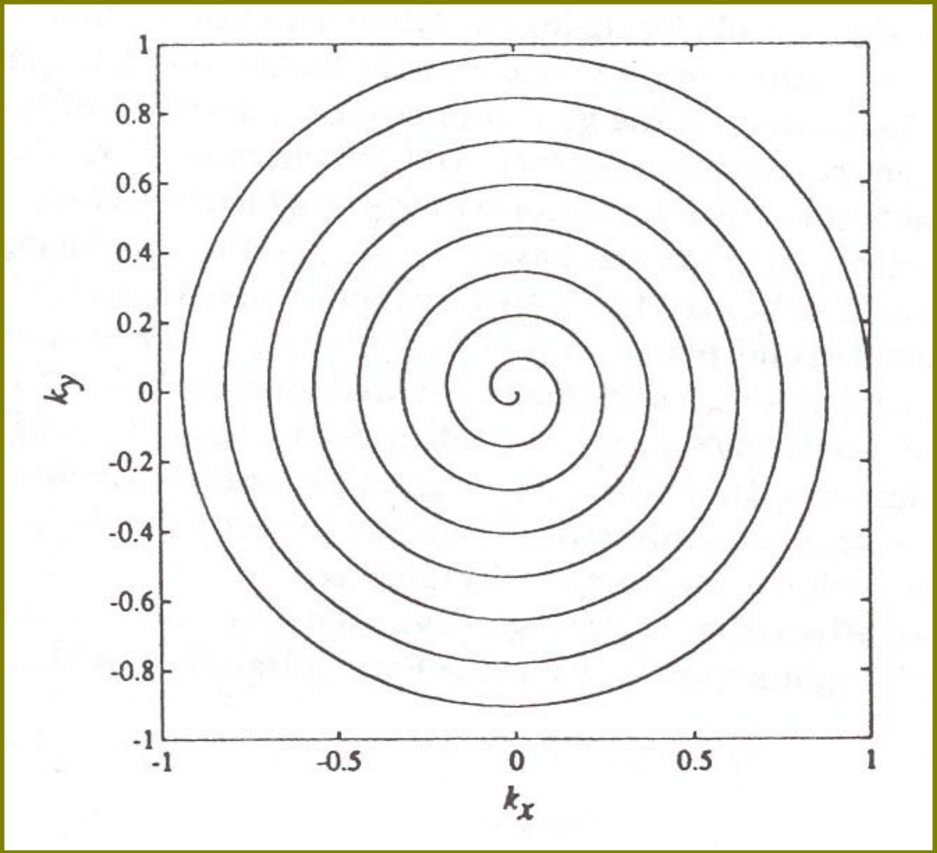



With constant slow gradient




With blipped echo planar gradient

# Spiral trajectory

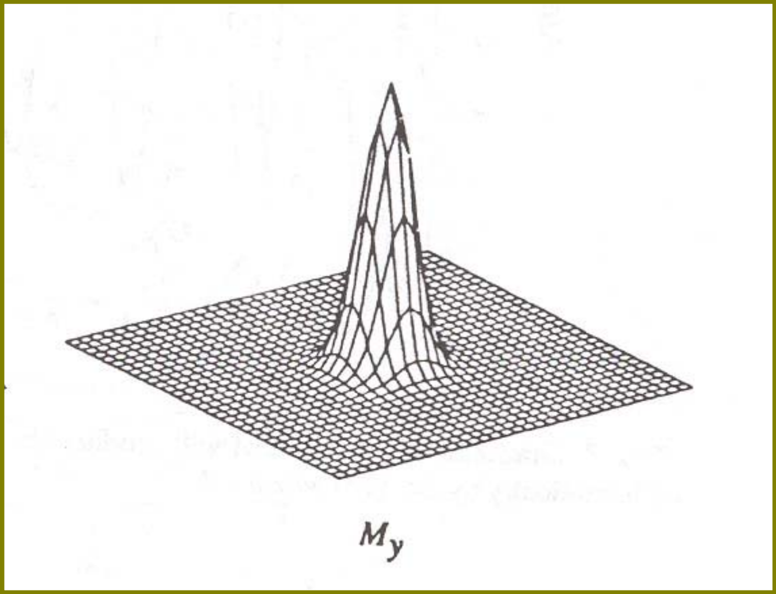


# Spiral trajectory

- $$k_x(t) = A(1-\frac{t}{T})\cos\frac{2\pi nt}{T}$$
  $$k_y(t) = A(1-\frac{t}{T})\sin\frac{2\pi nt}{T}$$
- T : pulse interval
- n : 轉了幾圈, 影響first sidelobe 的位置
- A : excitation範圍大小

# Design example

- Circularly symmetric Gaussian




$$D(\mathbf{k}) = \alpha\, e^{-\beta^2 (k_x^2 + k_y^2)/A^2}$$

# 2-D selective excitation

- 想要激發的範圍?強度分布?
- Fourier transform → D(**k**)
- Select W(**k**), S(**k**)
- W(**k**)*S(**k**) ~ D(**k**)
- S(**k**) → **G**(t)
- W(**k**) & **G**(t) → $B_1$(**t**)

# 幫大家回憶一下

- $W(\mathbf{k})$ : spatial weighting function

$$W(\mathbf{k}(t)) = \frac{B_1(t)}{\left|\gamma \mathbf{G}(t)\right|}$$

- $S(\mathbf{k})$ : unit weight trajectory

$$S(\mathbf{k}) = \int_0^T \left\{ {}^3\delta(\mathbf{k}(t) - \mathbf{k}) \left|\dot{\mathbf{k}}(t)\right| \right\} dt$$

- $\mathbf{k}(t) = -\gamma \int_t^T \mathbf{G}(s)\, ds$

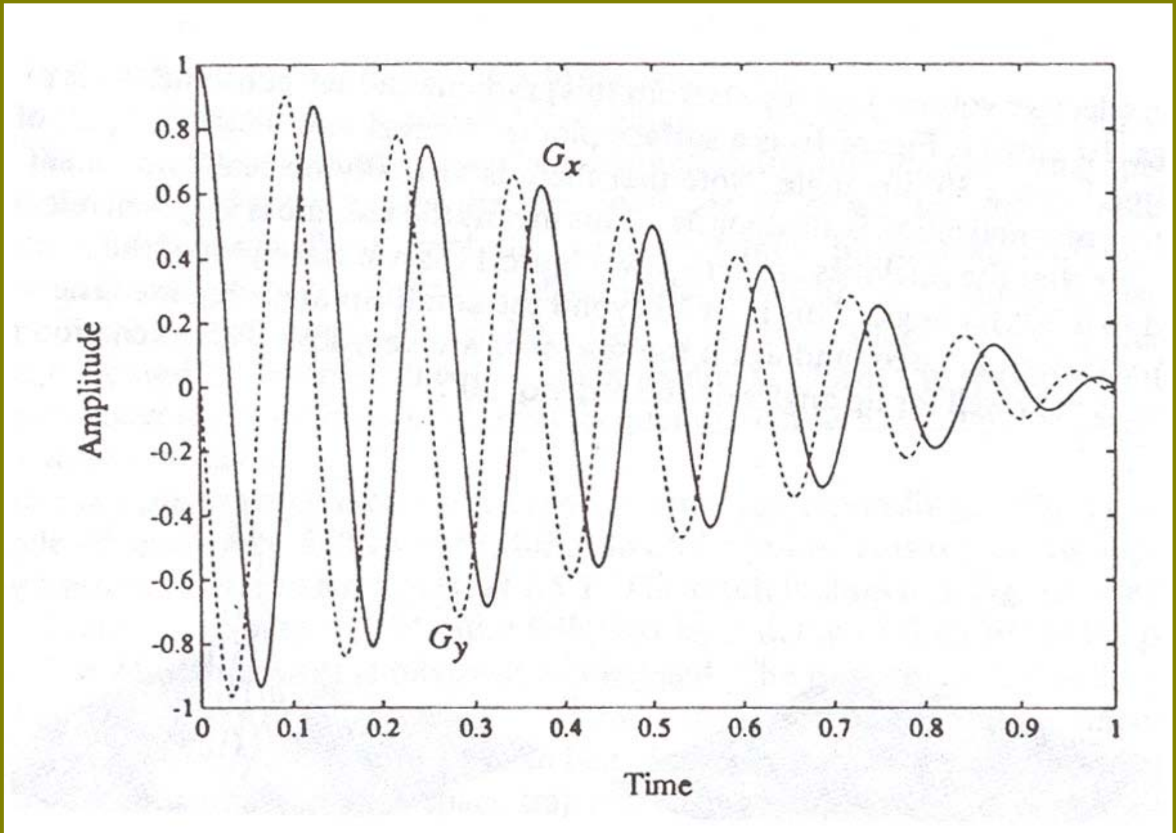
# 大家隨便看看就好

- $$G_x(t) = -\frac{A}{\gamma T}\left[2\pi n(1-\frac{t}{T})\sin\frac{2\pi nt}{T} + \cos\frac{2\pi nt}{T}\right]$$

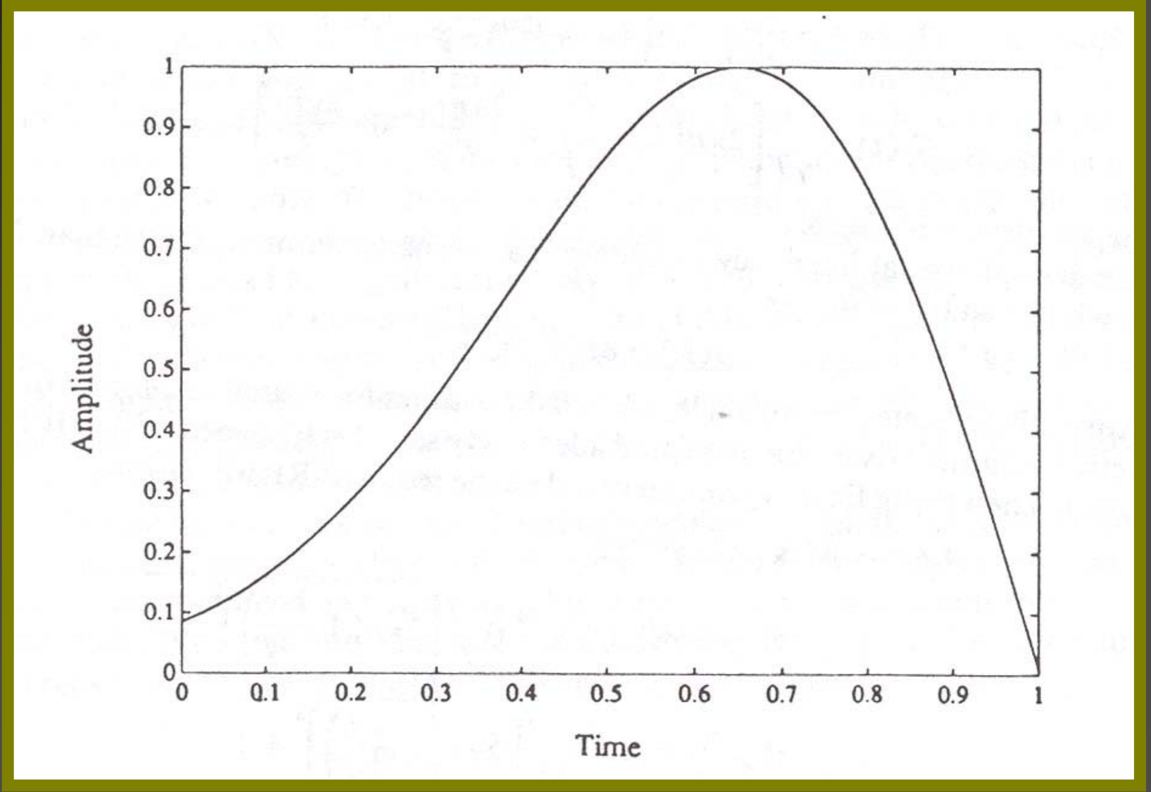
  $$G_y(t) = \frac{A}{\gamma T}\left[2\pi n(1-\frac{t}{T})\cos\frac{2\pi nt}{T} - \sin\frac{2\pi nt}{T}\right]$$

- $$B_1(t) = \gamma\alpha\frac{A}{T}e^{-\beta^2(1-t/T)^2}\sqrt{\left[2\pi n(1-\frac{t}{T})\right]^2+1}$$

# Gradient profile

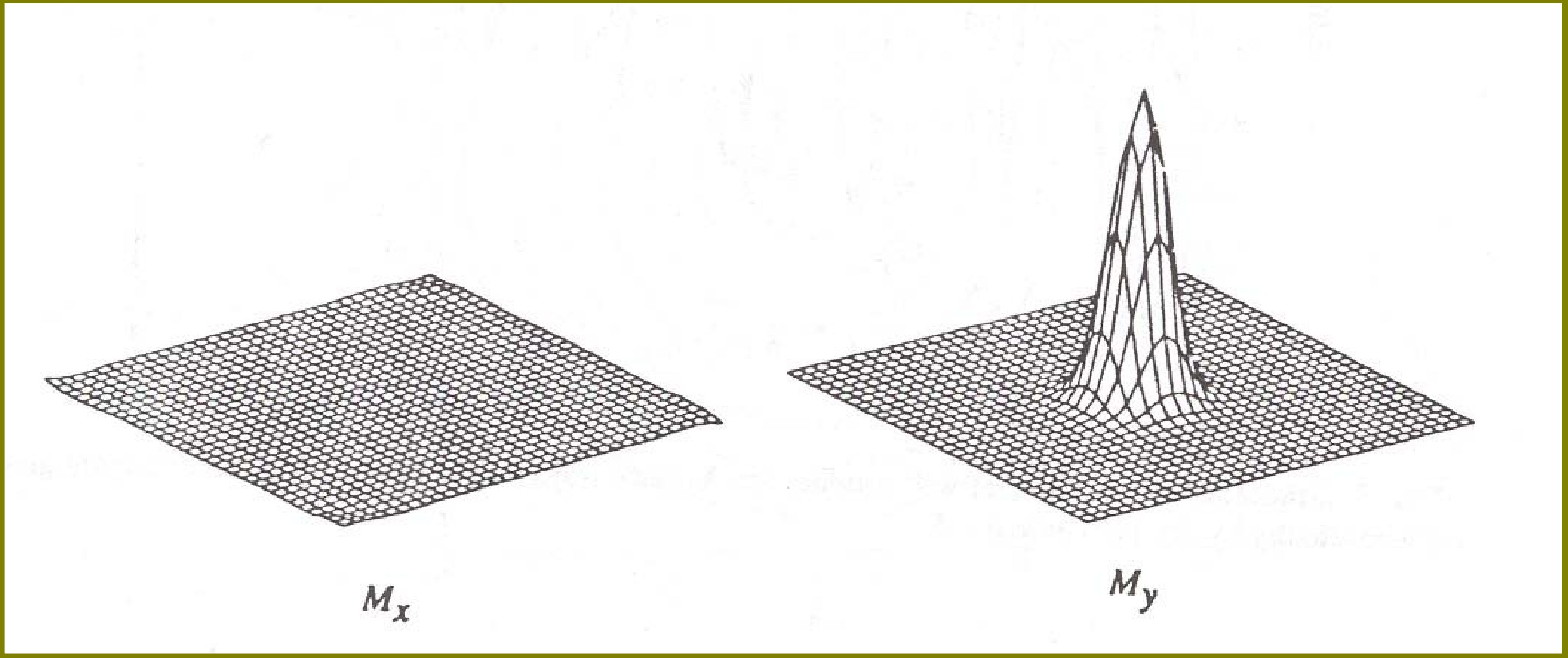


# RF profile

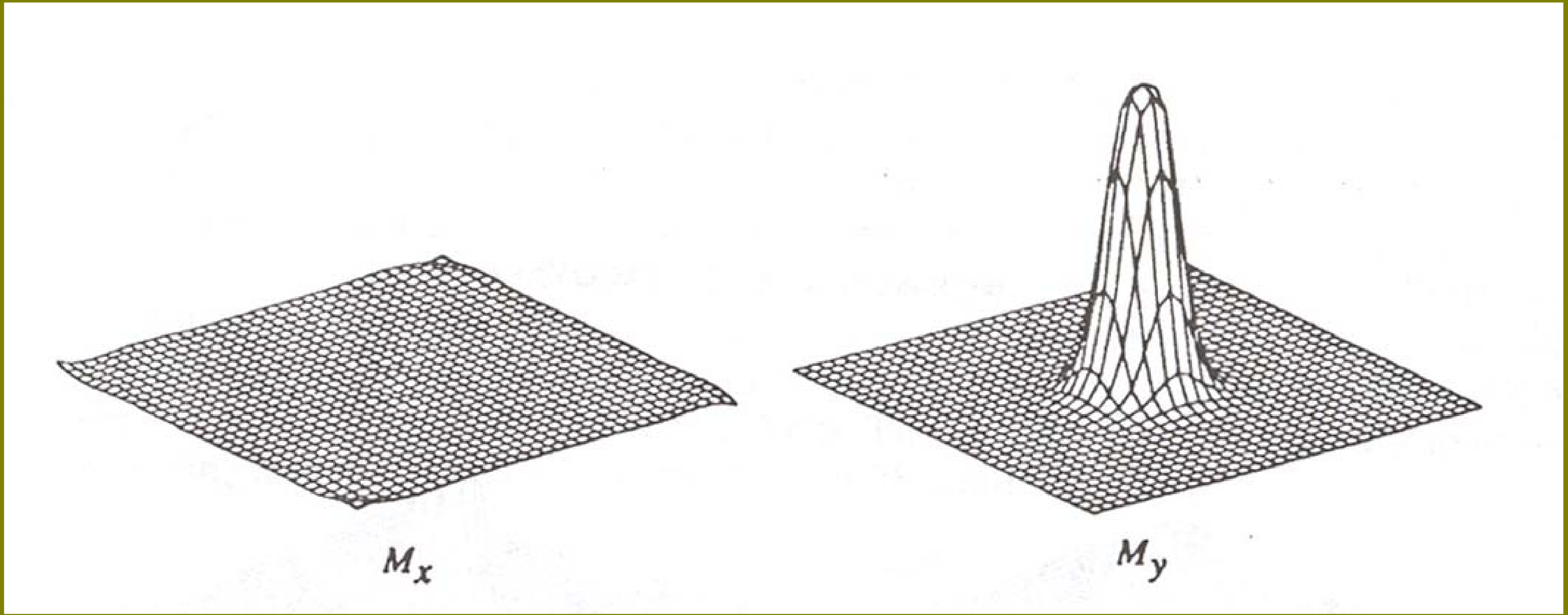


1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Amplitude
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Time

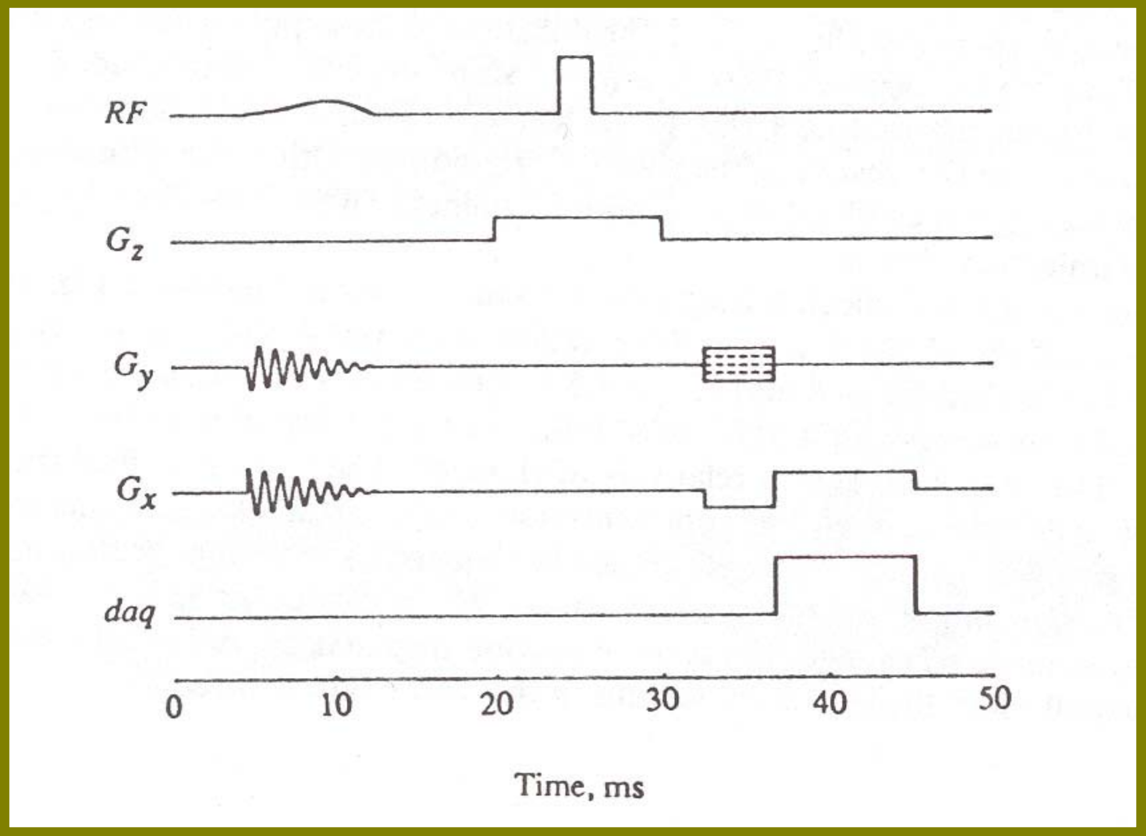
# Simulation -- 30°



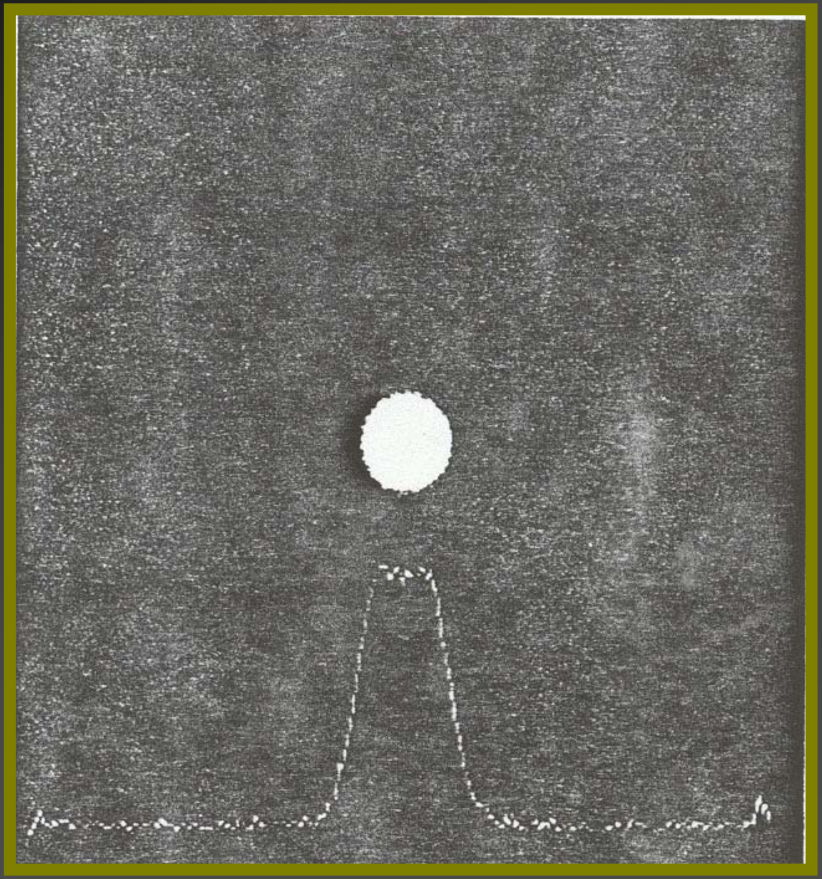
# Simulation -- 90°



# Pulse sequence

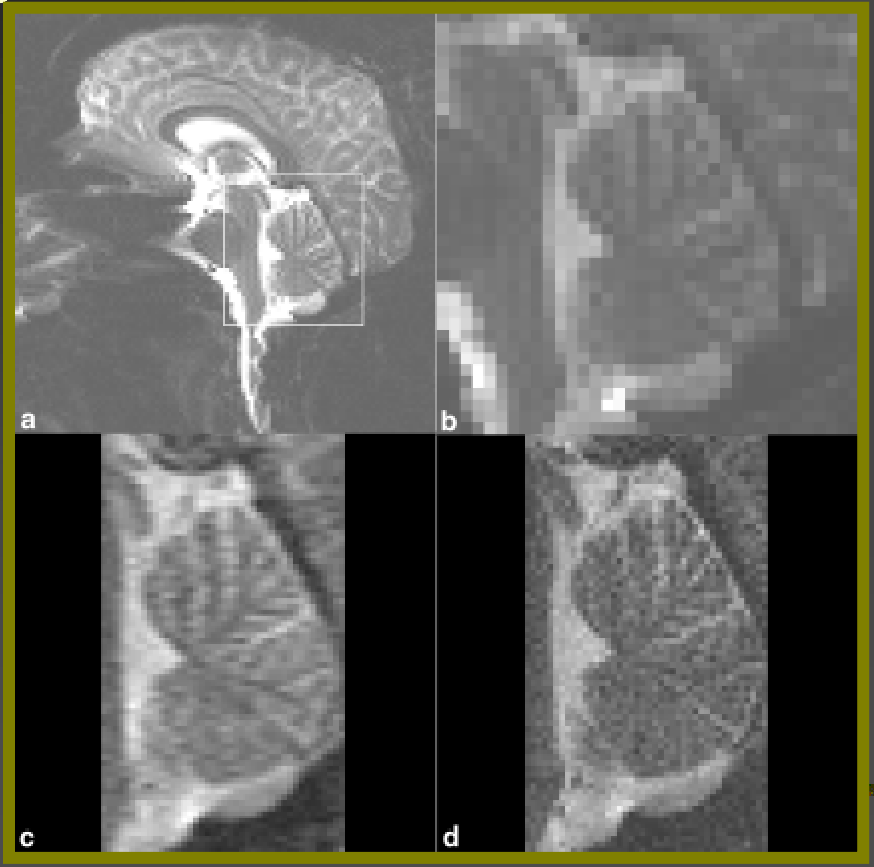


# Phantom image



# Some fresher images

- Rieseberg et al. 2002



# 休息乎?

# Spatial-spectral pulse

- 激發slice中某些特定頻率的proton
- 跟前面講的概念是差不多的
  - Spatial-spatial
  - Spatial-spectral

# K-space trajectory

- K-space
  - kx-ky
  - kz-kw
- $$\mathbf{k}_z(t) = -R\int_t^T \mathbf{G}(s)\,ds$$

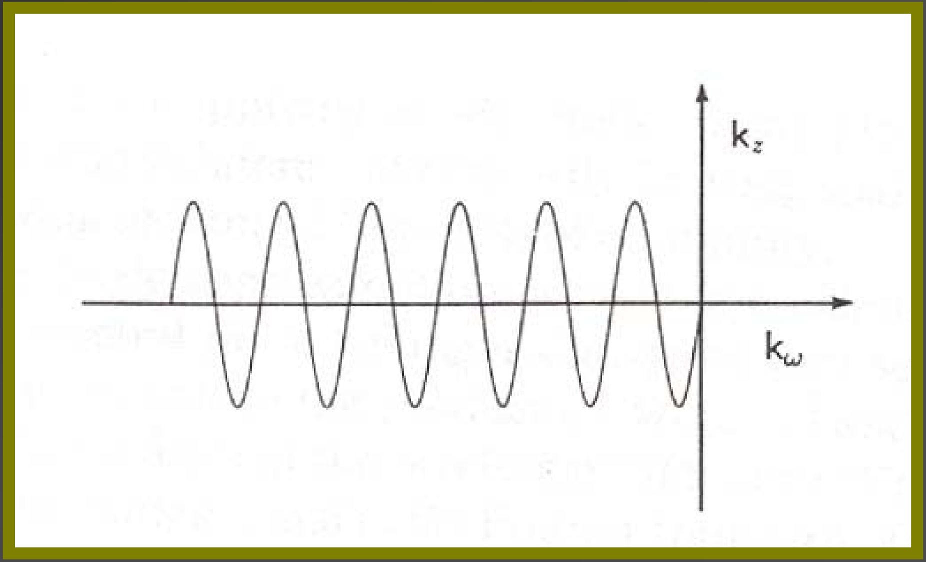
  $$\mathbf{k}_\omega(t) = t - T$$

# K-space trajectory

- Sinusoidal oscillation

$$G_z(t) = G\cos\Omega(t - T)$$

$$k_z(t) = \frac{RG}{\Omega}\sin\Omega k_\omega$$



# 偷懶一下…

- $M_{xy}$跟$B_{1,}$ $G_z$的關係
- Well, it’s a long story …
- 以前面的waveform為例

$$M_{xy}(z,\omega) = i2\pi\gamma M_0 F^{-1}\{W(k_w)\} \times \delta(k_z)$$

$$* \sum_{n=-\infty}^{\infty} [F^{-1}\{W(k_z)\} * \frac{J_1(Az')}{2Az'} * J_n(z')]\delta(w - n\Omega)$$

# 需要決定什麼?

- K-space trajectory
- $G_z$'s frequency & amplitude
  - Position of sidelobe
- RF envelope
  - K-space weighting & slice profile
- Pulse length

# 來看例子吧

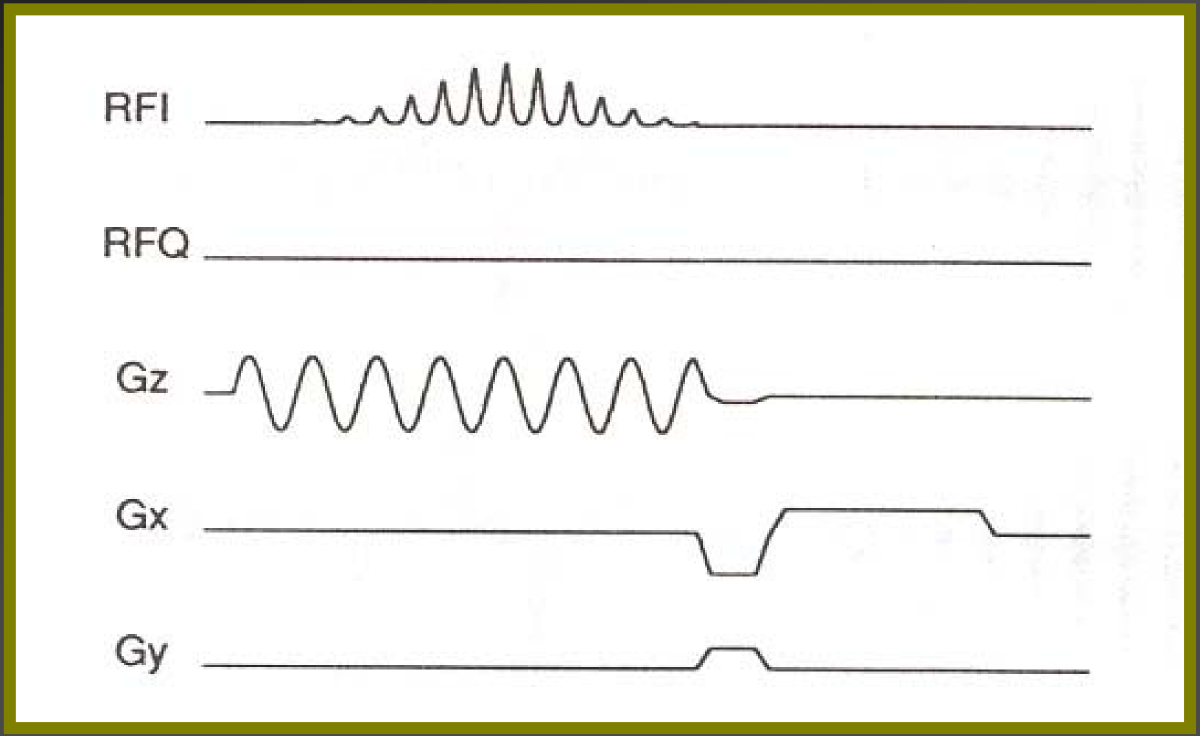
- Excite water, suppress fat
- 頻率相差230Hz in 1.5 T
- Water → main lobe
- Fat → null between ✓main lobe & 1st sidelobe
  1st sidelobe & 2nd sidelobe
- $G_z$ frequency = 460Hz

# 還有一些參數

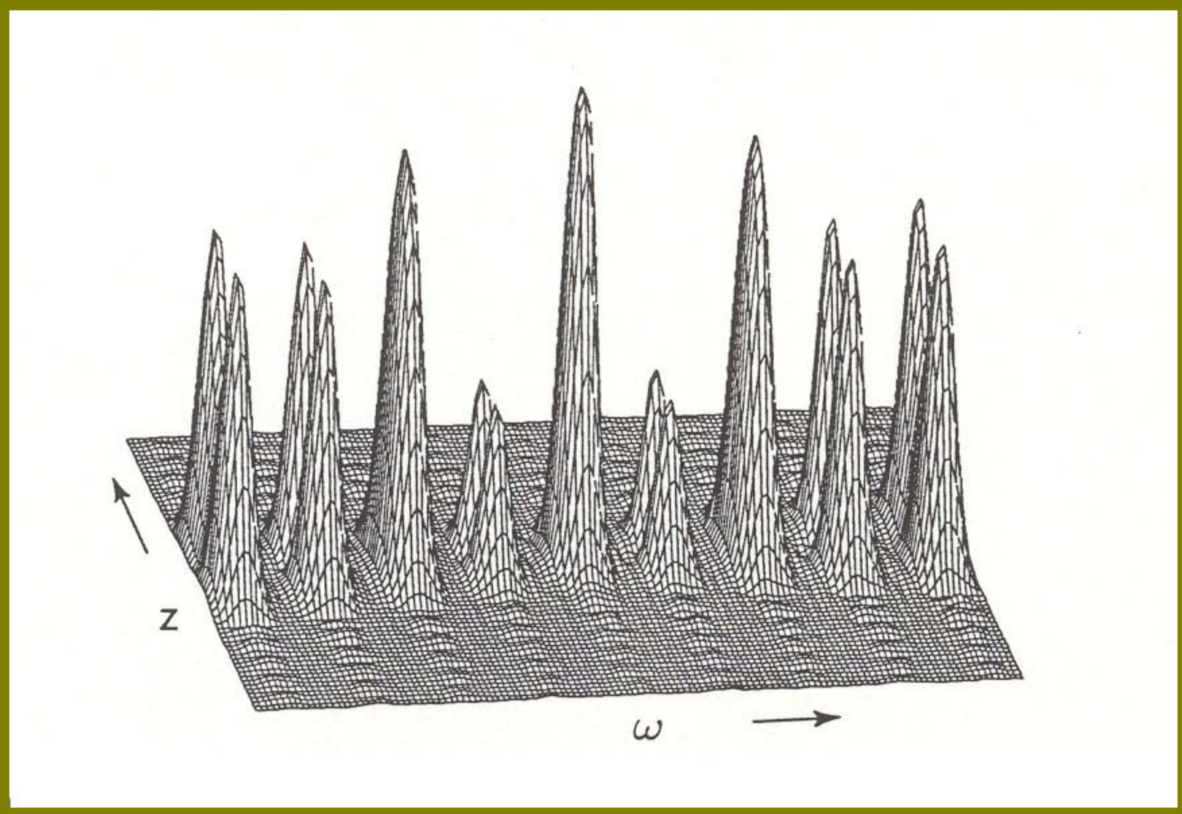
- Gaussian weighting
- $G_z(t) = G\cos\Omega(t-T)$

  $B_1(t) = B_1 e^{-\pi[\sin\Omega(t-T)/U]^2} e^{-\pi[(t-T/2)/V]^2} \cos\Omega(t-T)$
- T U V G : 適當就好
  - T ↑: transmission band ↓
  - G ↑ : slice width ↓
  - U,V : slice profiles

# Pulse sequence

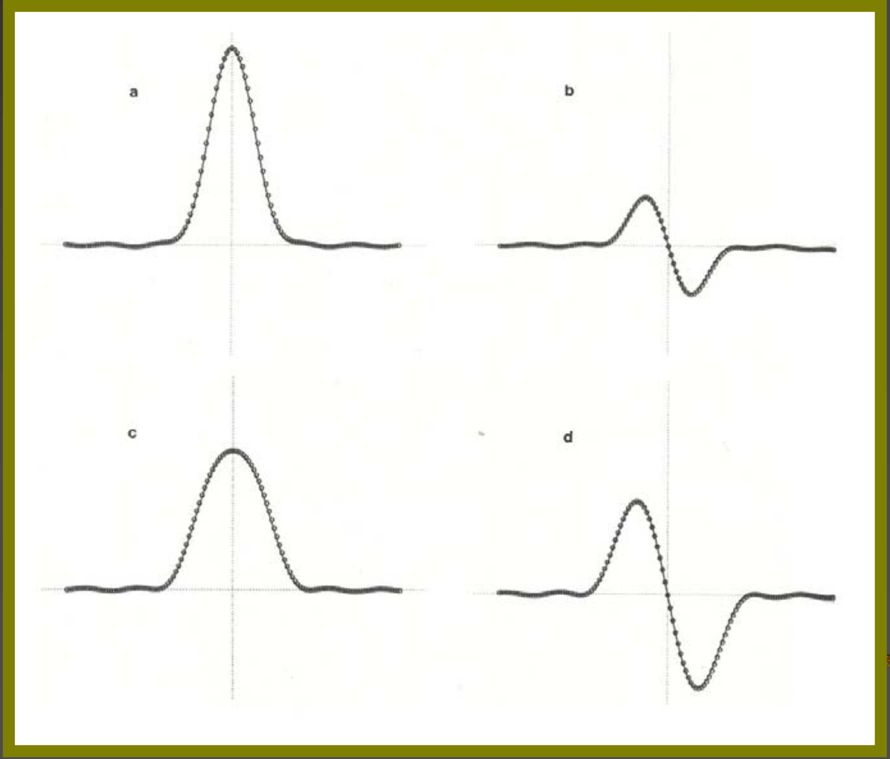


# Simulation



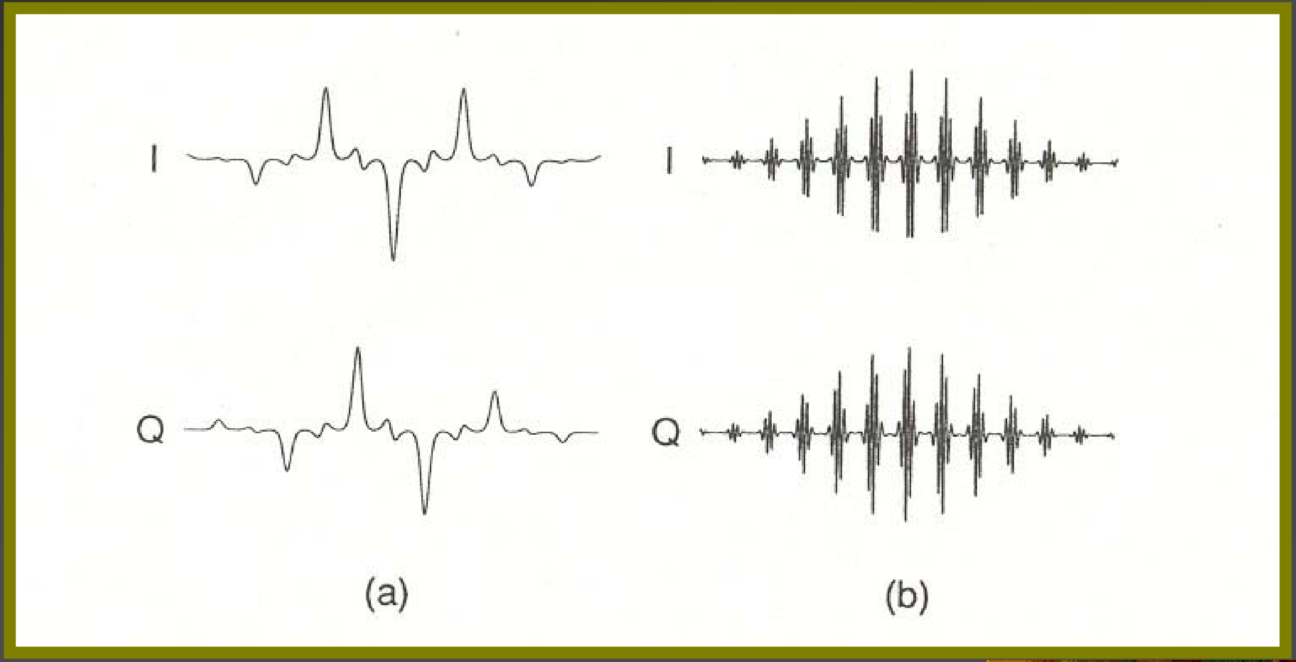
# Sidelobes along w axis

- 第2n個：even function
- 第2n+1個：odd function

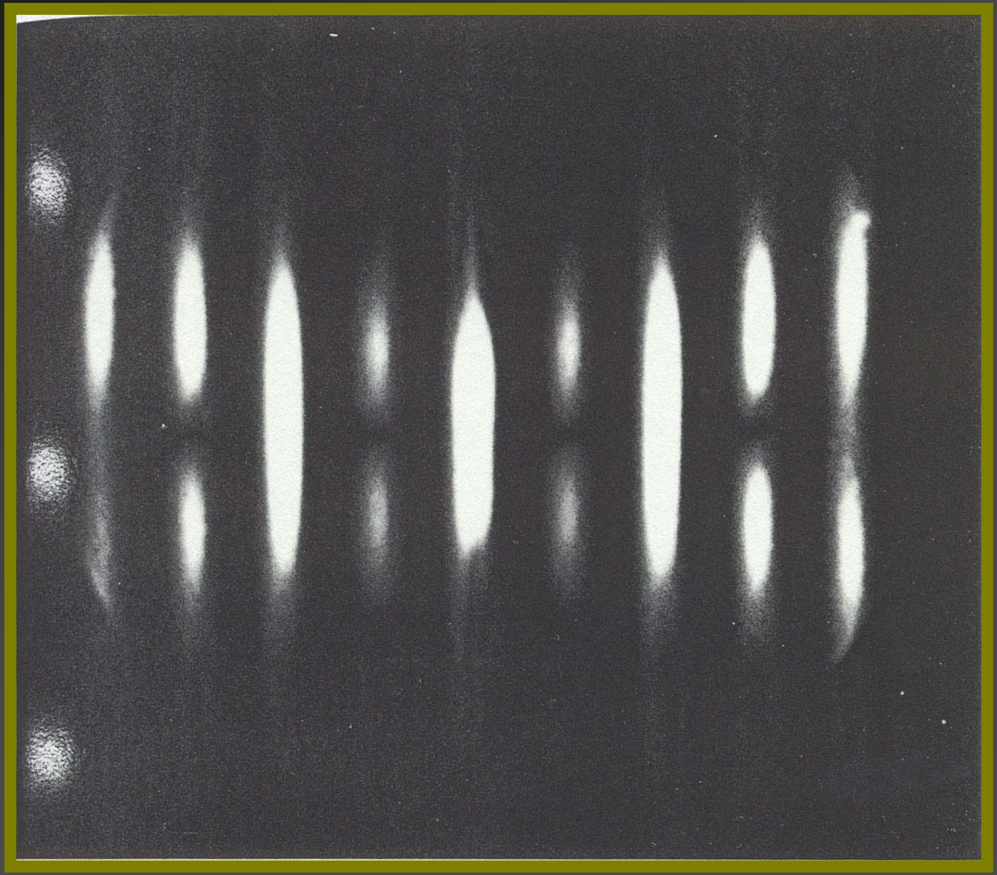


# If shift is needed…

- Frequency shift Δω → $B_1 \cdot e^{i\Delta\omega t}$
- Spatial shift Δz → $B_1 \cdot e^{ik_z(t)\Delta z}$

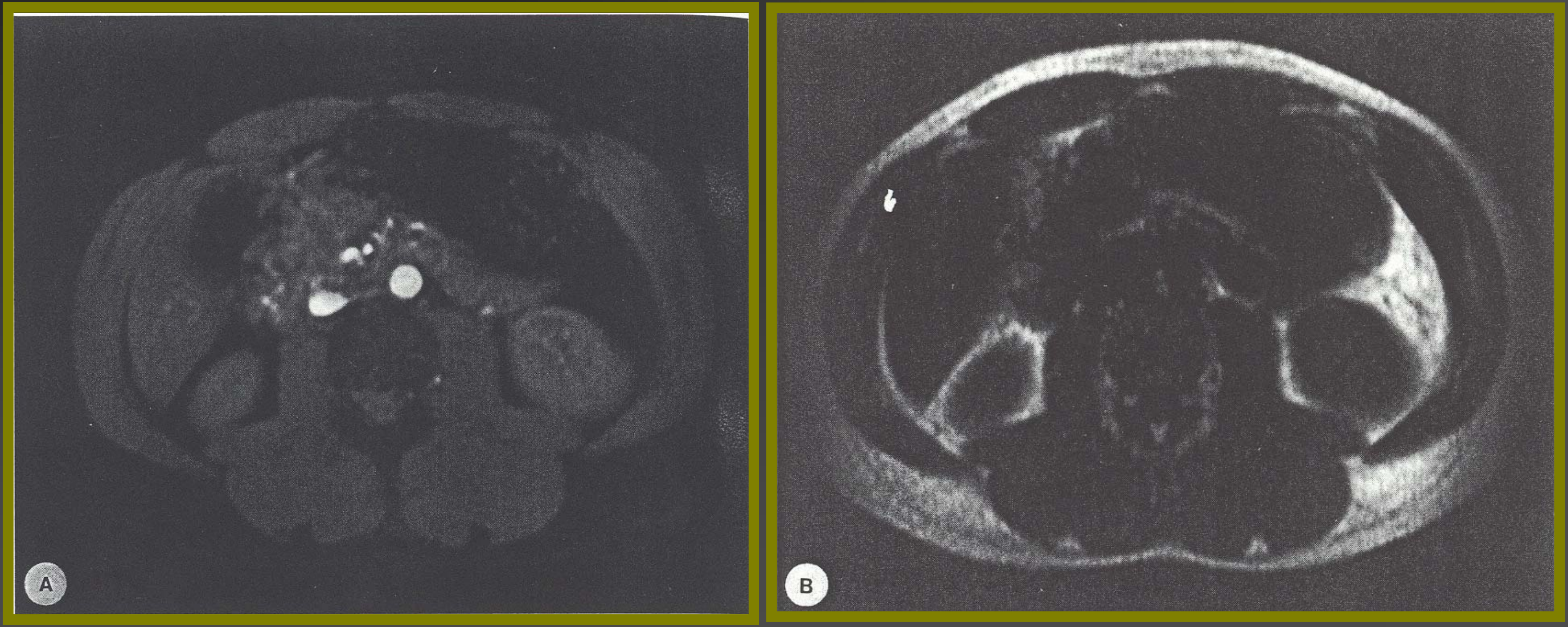


# Experiment results



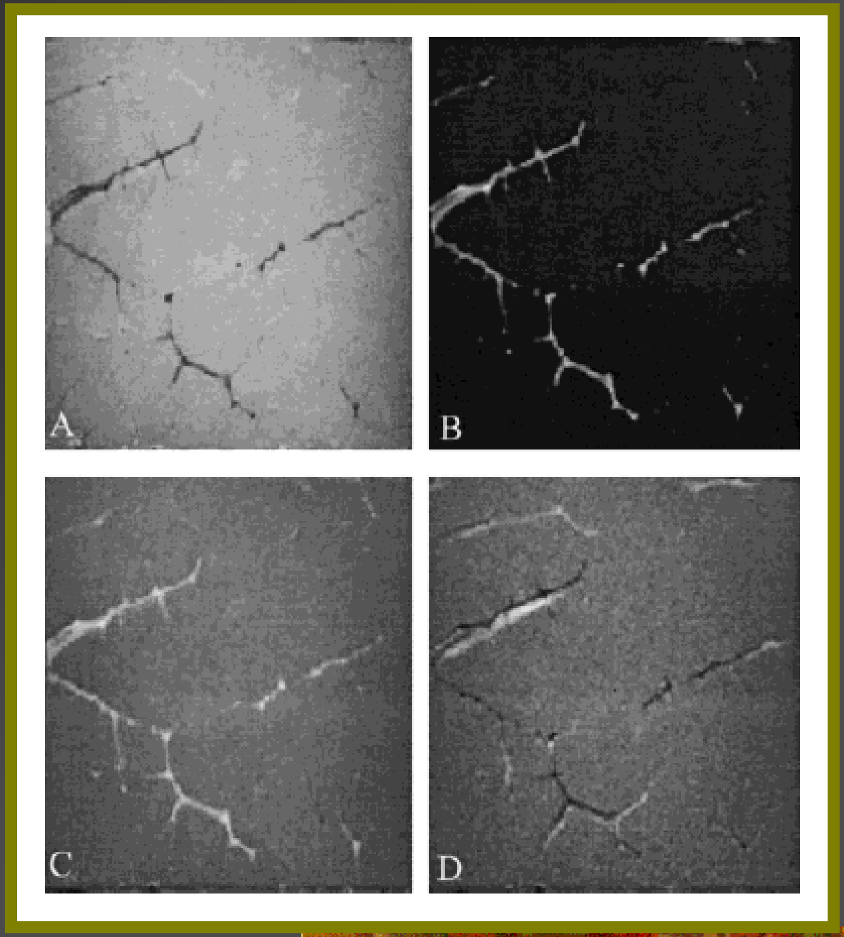
# Head images


# Abdominal images



# 較新的影像

- Laurent et al., 2000

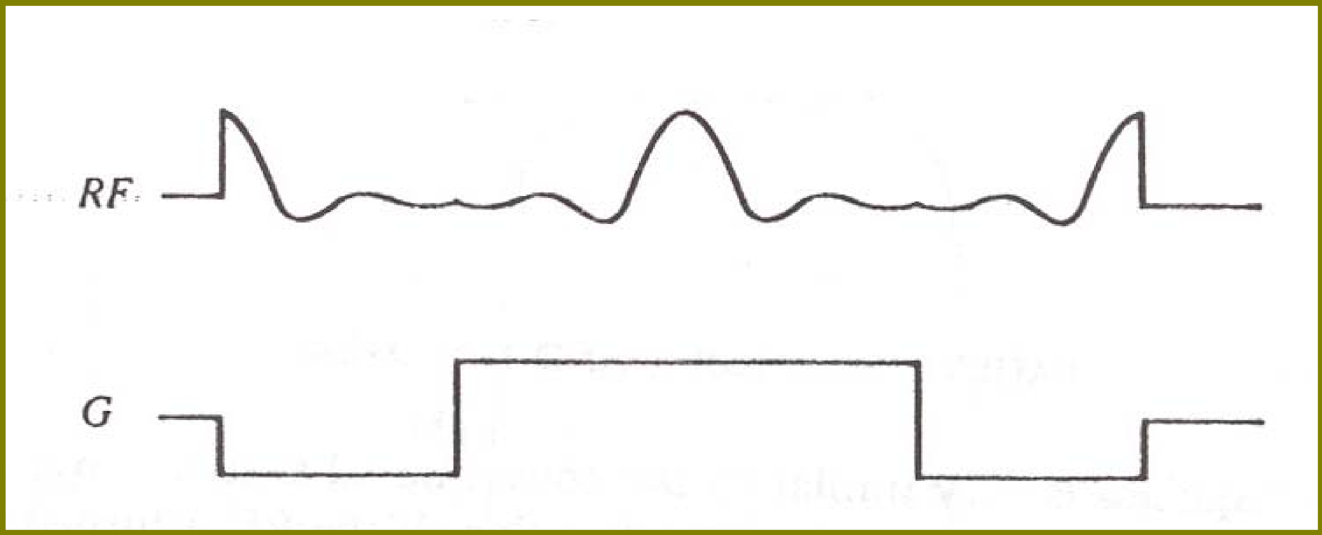


# References

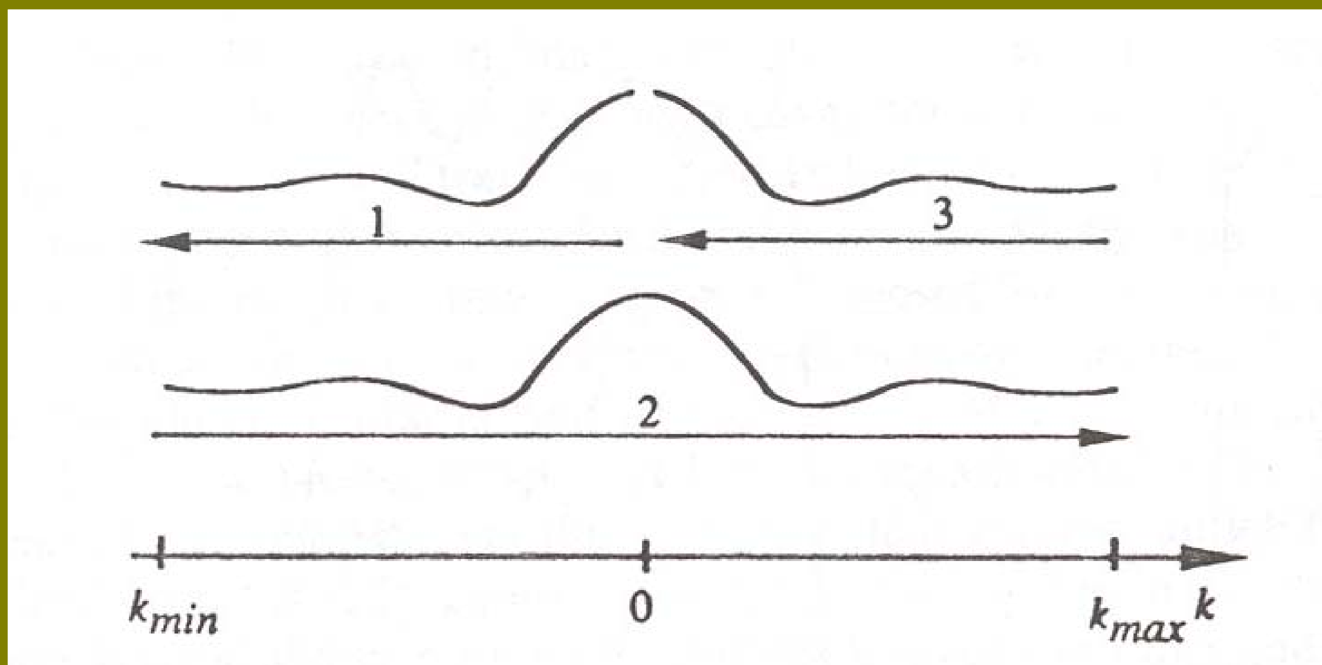

- John Pauly, Dwight Nishimura, Albert Macovski, “A k-space analysis of small-tip-angle excitation”, JMR 81:43-56, 1989
- Craig H. Meyer, John M. Pauly, Albert Macovski, Dwight G. Nishimura, “Simutaneous spatial and spectral selective excitation”, MRM 15:287-304, 1990

- Susanne Rieseberg, Jens Frahm, Jurgen Finsterbusch, “Two-dimensional spatially-selective RF excitation pulses in echo-planar imaging”, MRM 47: 1186-93, 2002
- W.M. Laurent, J.M. Bonny, J.P. Renou, “Imaging of water and fat fractions in high-field MRI with multiple slice chemical shift-selective inversion recovery”, JMRI 12:488-496, 2000

# Inherently refocus pulse

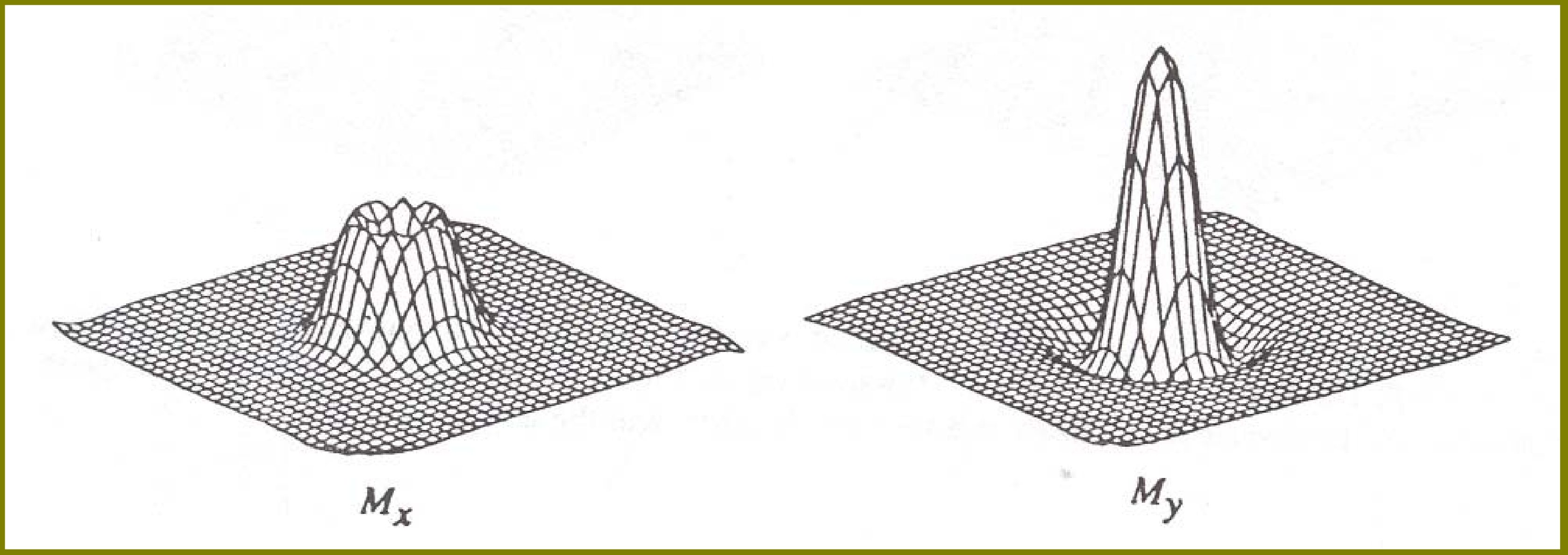


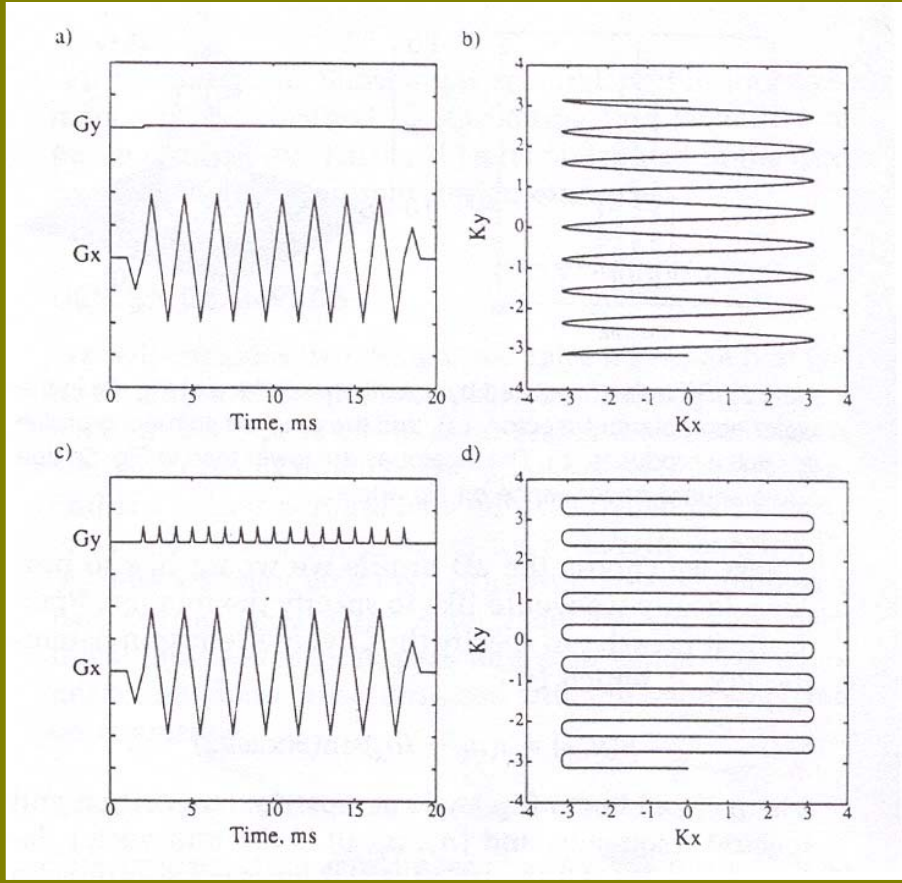
# Inherently refocus pulse



# Simulation -- off resonance

- Half cycle off resonance





a)
b)
c)
d)
Gy
Gx
Time, ms
Ky
Kx