Shinnar-Le Roux RF Pulse Design

莊子肇

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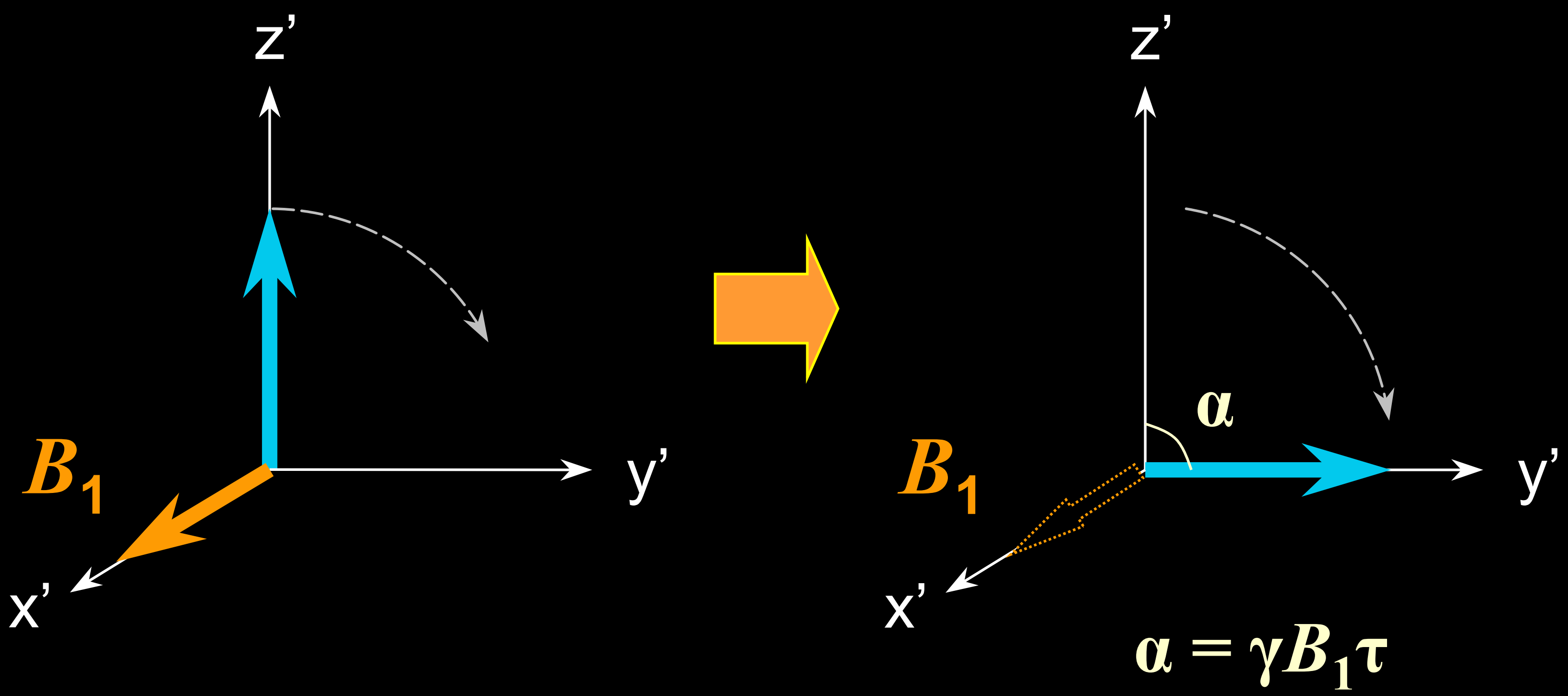
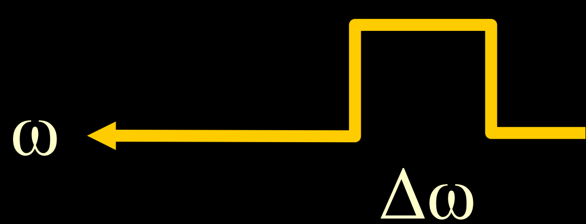
Review: RF Pulse



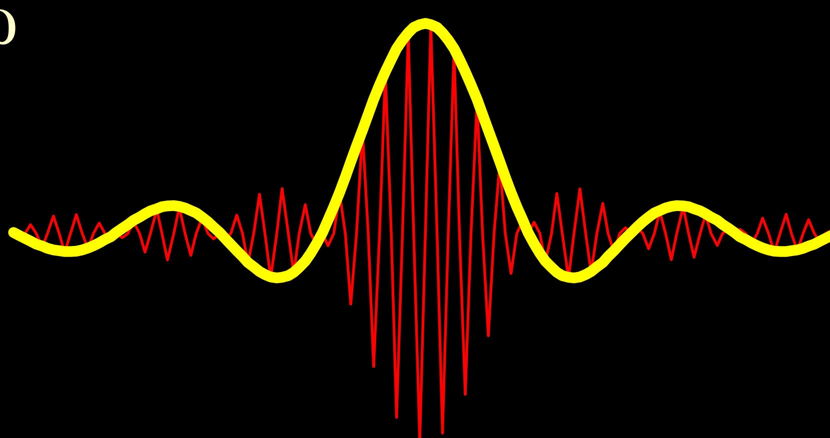
Review: Slice Selection

1. Choose a Δx slice.

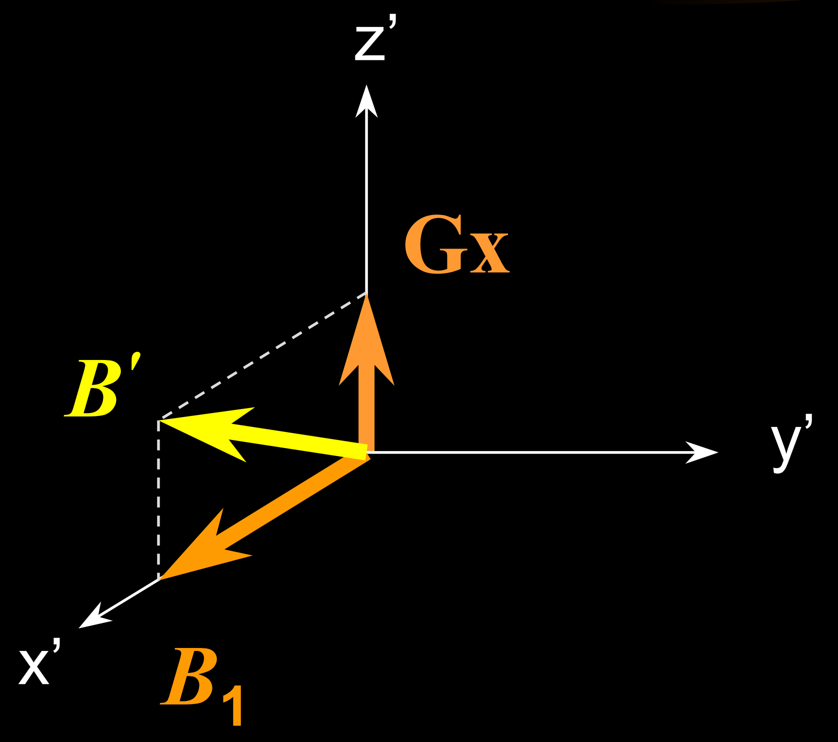


2. Correspond to $\Delta\omega$.

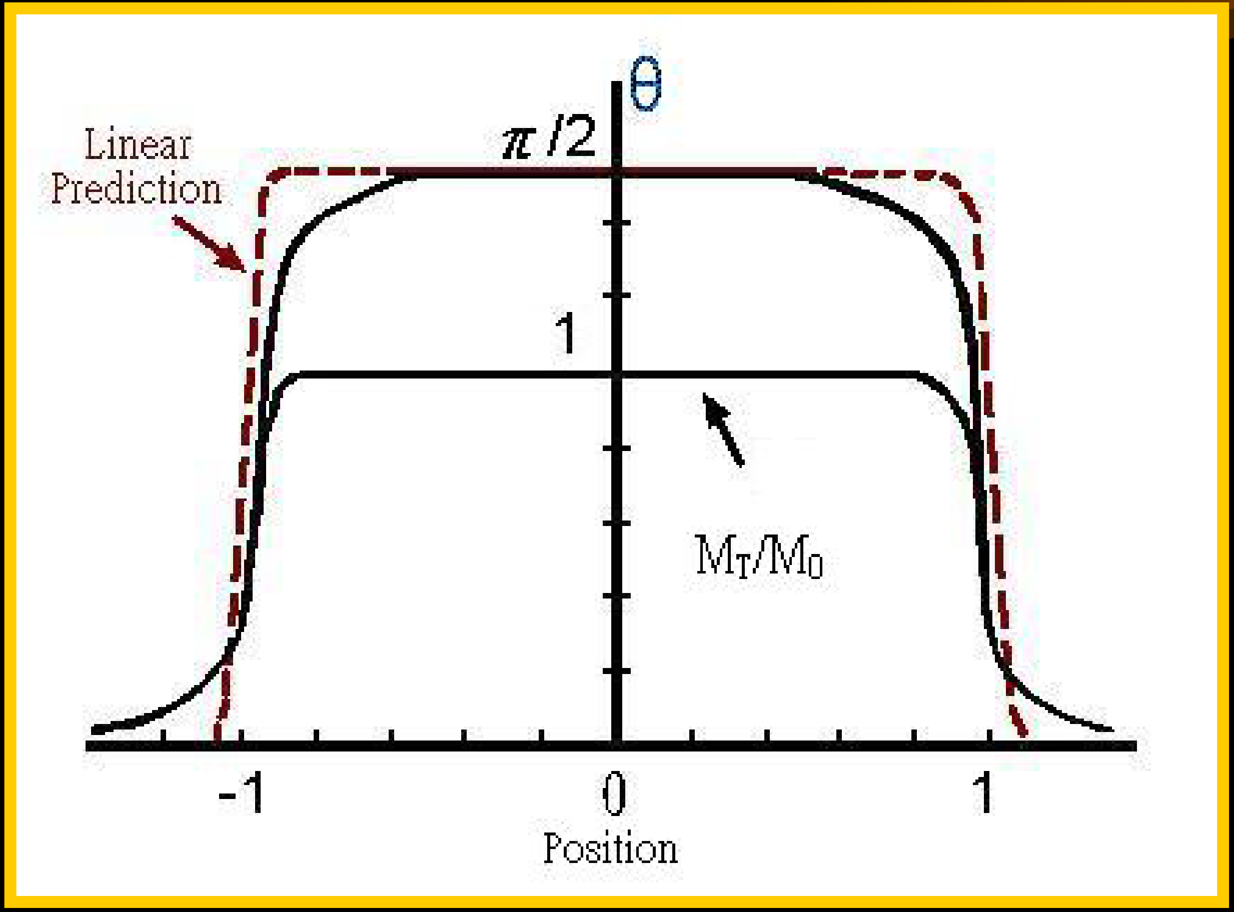
3. B_1 profile



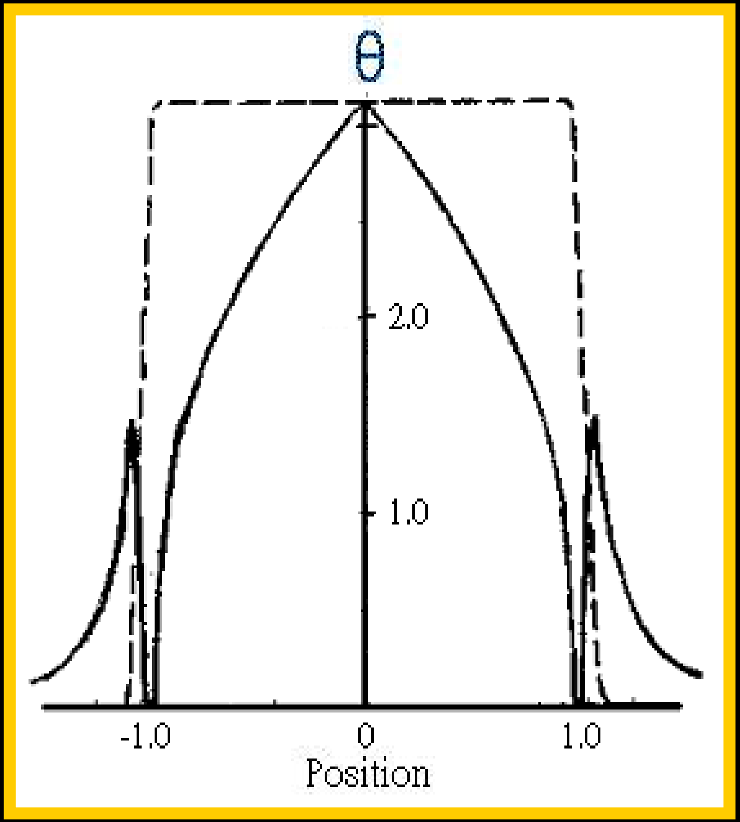
But in Practical,…..

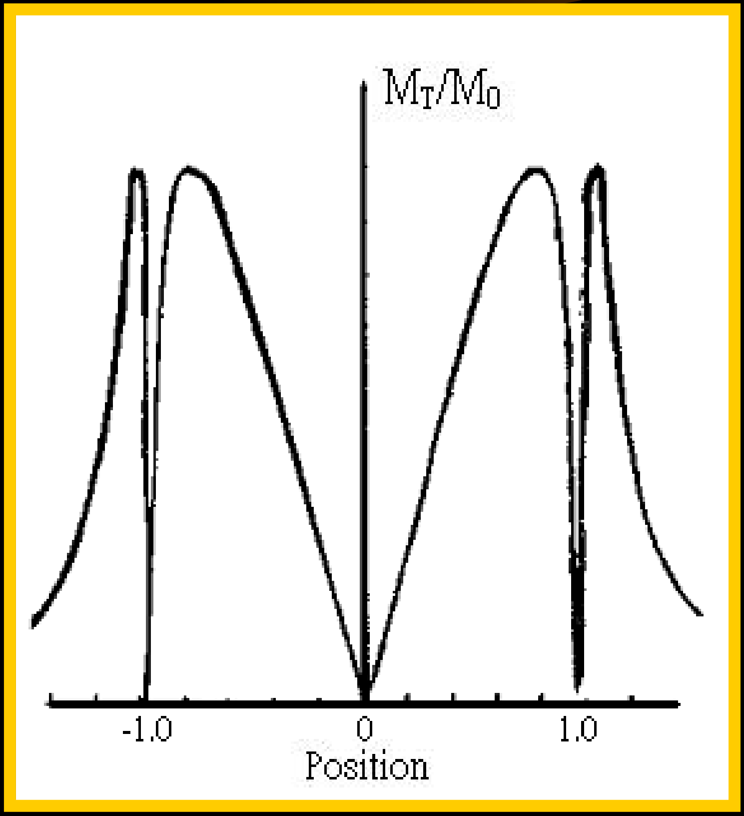


實際上(90º pulse)



實際上(180º pulse)





Does it matter?

- B_1: Given $\alpha = \pi/2$, τ =1ms

  $B_1 \sim 0.06$ Gauss

- Gx: Given G= 1 Gauss/cm, x= 0.5 cm

  Gx~ 0.5 Gauss !!

Yes!!

Shinnar-Le Roux Algorithm

$$B_1(t) \xleftrightarrow{SLR} M(x,t)$$

- Neglecting relaxation
- Non-linear Bloch Equation
- Piece-wise constant approximation RF

Bloch Equation

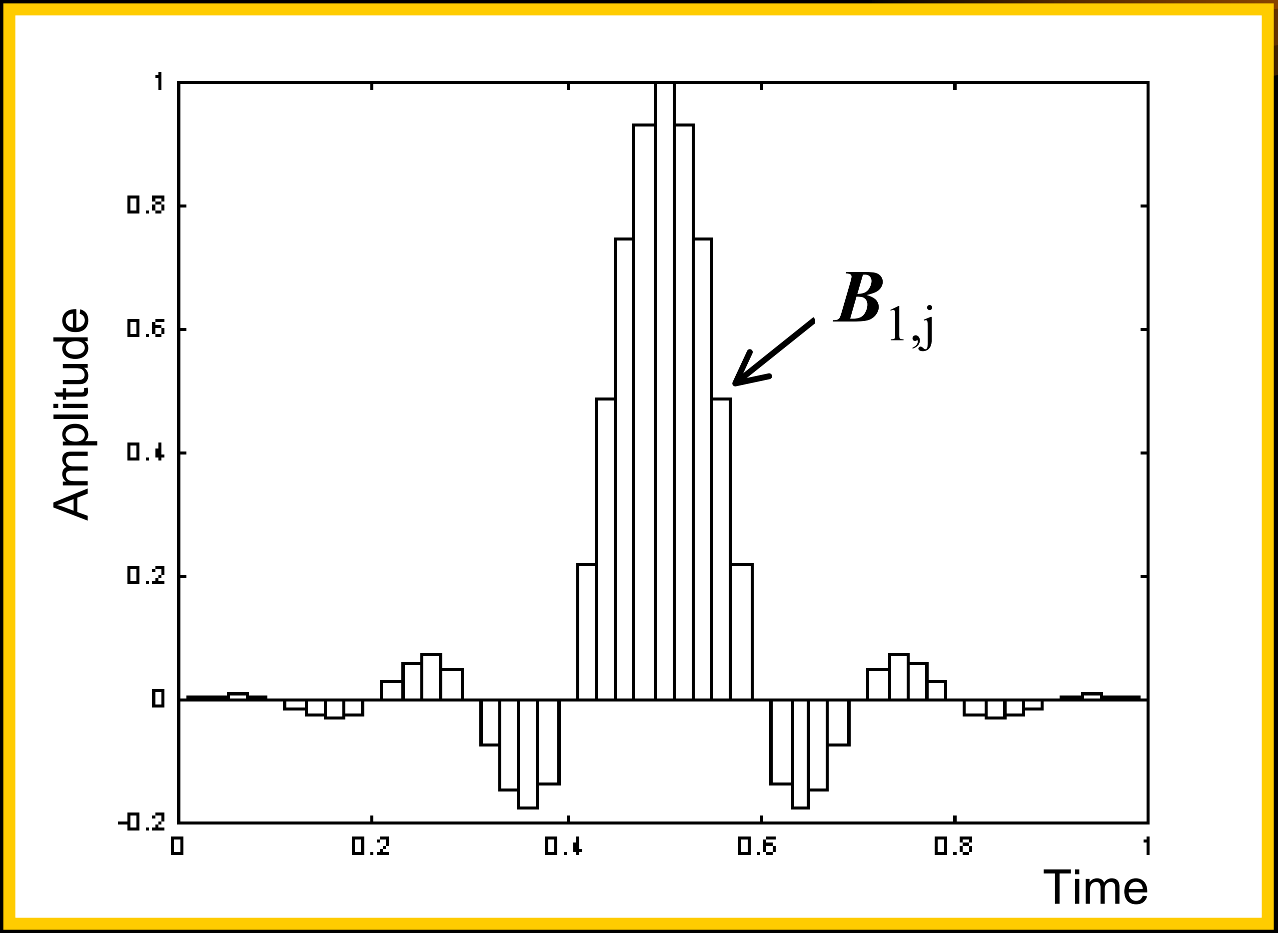
$$\left(\frac{\partial \vec{\mathrm{M}}}{\partial \mathrm{t}}\right)_{\mathrm{rot}} = \gamma \vec{\mathrm{M}} \times \vec{B}' \quad , \quad \vec{B}' = \left(B_{1,x}, B_{1,y}, \mathrm{Gx}\right)$$

$$\begin{pmatrix} \dot{M}_x \\ \dot{M}_y \\ \dot{M}_z \end{pmatrix}_{rot} = \gamma \begin{pmatrix} 0 & Gx & -B_{1,y} \\ -Gx & 0 & B_{1,x} \\ B_{1,y} & -B_{1,x} & 0 \end{pmatrix} \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix}_{rot}$$

Shinnar-Le Roux Algorithm

- Neglecting relaxation
- Non-linear Bloch Equation
- Piece-wise constant approximation RF

RF Discrete Approximation



Shinnar-Le Roux Algorithm

$$B_1(t) \overset{SLR}{\longleftrightarrow} M(x,t)$$

Nonlinear Bloch Eq.
3*3 matrix

→

Spin Domain Representation
2*2 matrix

→

2 Polynomials

Spin Domain Representation

- At the jth step, $B_{1,j} = B_{1,x,j} + iB_{1,y,j}$ for Δt
  - Rotation angle

$$\phi_j = -\gamma \Delta t \sqrt{\left|B_{1,j}\right|^2 + (Gx)^2}$$

  - Axis of the rotation

$$\vec{n}_j = \frac{\gamma \Delta t}{\left|\phi_j\right|}\left(B_{1,x,j}, B_{1,y,j}, Gx\right)$$

Spin Domain Matrix

Let
$$p_j = \cos\phi_j/2 - i n_{z,j}\sin\phi_j/2$$
$$q_j = -i(n_{x,j} + n_{y,j})\sin\phi_j/2$$

at the j-th step
$$Q_j = \begin{pmatrix} p_j & -q_j^* \\ q_j & p_j^* \end{pmatrix}, \quad p_j p_j^* + q_j q_j^* = 1$$

for total rotation
$$Q = Q_n Q_{n-1} \cdots Q_1 = \begin{pmatrix} \alpha & -\beta^* \\ \beta & \alpha \end{pmatrix}$$

Spin Domain and Bloch Equation

$$\begin{pmatrix} M_{xy}^{+} \\ M_{xy}^{+\,*} \\ M_{z}^{+} \end{pmatrix} = \begin{pmatrix} \left(\alpha^{*}\right)^{2} & -\beta^{2} & 2\alpha^{*}\beta \\ -\left(\beta^{*}\right)^{2} & \alpha^{2} & 2\alpha\beta^{*} \\ -\alpha^{*}\beta^{*} & -\alpha\beta & \alpha\alpha^{*}-\beta\beta^{*} \end{pmatrix} \begin{pmatrix} M_{xy}^{-} \\ M_{xy}^{-\,*} \\ M_{z}^{-} \end{pmatrix}$$

$$M_{xy} = M_{x} + iM_{y}$$

Forward SLR Transform

- According to hard pulse approximation, small flip angle rotation can be modeled by two sequential processes.
  1. By local gradient field: $-\gamma G x \Delta t$
  2. By applied RF: $-\gamma B_1 \Delta t$

Two Sequential Spin Matrices

$$Q_j = \begin{pmatrix} C_j & -S_j^* \\ S_j & C_j \end{pmatrix} \begin{pmatrix} z^{1/2} & 0 \\ 0 & z^{-1/2} \end{pmatrix}$$

$$C_j = \cos(\gamma \left| B_{1,j} \right| \Delta t / 2)$$

$$S_j = i e^{i \angle B_{1,j}} \sin(\gamma \left| B_{1,j} \right| \Delta t / 2)$$

$$z = e^{i \gamma G x \Delta t}$$

State-Space Description

$$\begin{pmatrix} \alpha_j & -\beta_j^* \\ \beta_j & \alpha_j^* \end{pmatrix} = Q_j \begin{pmatrix} \alpha_{j-1} & -\beta_{j-1}^* \\ \beta_{j-1} & \alpha_{j-1}^* \end{pmatrix}$$

$$\begin{pmatrix} \alpha_j \\ \beta_j \end{pmatrix} = z^{1/2} \begin{pmatrix} C_j & -S_j^* \\ S_j & C_j \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & z^{-1} \end{pmatrix} \begin{pmatrix} \alpha_{j-1} \\ \beta_{j-1} \end{pmatrix}$$

$$\alpha_j \alpha_j^* + \beta_j \beta_j^* = 1$$

天啊！還有多少數學？

Define $A_j = z^{j/2}\alpha_j$

$B_j = z^{j/2}\beta_j$

then $A_n(z) = \sum_{j=0}^{n-1} a_j z^{-j}$

$B_n(z) = \sum_{j=0}^{n-1} b_j z^{-j}$

where $z^{-1} = e^{-i\gamma G x \Delta t}$

and a_j、b_j are constants

satisfy $|A_n|^2 + |B_n|^2 = 1$

整理一下

- $A_n(z)$、$B_n(z)$到底代表什麼？
  - $A_n(z)$ and $B_n(z)$ can be used to derive the slice profile and in-slice phase through well-known digital filter algorithms.
  - We'll check it out later!

Inverse SLR Transform

$$\begin{pmatrix} A_j \\ B_j \end{pmatrix} = \begin{pmatrix} C_j & -S_j^* z^{-1} \\ S_j & C_j z^{-1} \end{pmatrix} \begin{pmatrix} A_{j-1} \\ B_{j-1} \end{pmatrix}$$

$$\Downarrow$$

$$\begin{pmatrix} A_{j-1} \\ B_{j-1} \end{pmatrix} = \begin{pmatrix} C_j & S_j^* \\ -S_j z & C_j z \end{pmatrix} \begin{pmatrix} A_j \\ B_j \end{pmatrix} = \begin{pmatrix} C_j A_j + S_j^* B_j \\ z\left(-S_j A_j + C_j B_j\right) \end{pmatrix}$$

Backward Recursion

- Let $A_{j,k}$, $B_{j,k}$ be the coefficients of the z^{-k} term of A_j and B_j.

$$-S_j A_{j,0} + C_j B_{j,0} = 0$$

$$\Rightarrow \frac{B_{j,0}}{A_{j,0}} = \frac{S_j}{C_j} = \frac{ie^{i\angle B_{1,j}} \sin\left(\gamma \left|B_{1,j}\right| \Delta t / 2\right)}{\cos\left(\gamma \left|B_{1,j}\right| \Delta t / 2\right)}$$

$$\Rightarrow B_{1,j} = \frac{2}{\gamma \Delta t} \tan^{-1} \left|\frac{B_{j,0}}{A_{j,0}}\right| \cdot \angle\left(\frac{-iB_{j,0}}{A_{j,0}}\right)$$

神奇嗎？

$$B_1(t) \xleftrightarrow{\textbf{\textit{SLR}}} (A_n(z), B_n(z))$$

- Unique invertible transform relationship between an RF pulse and two polynomials $A_n(z)$ and $B_n(z)$

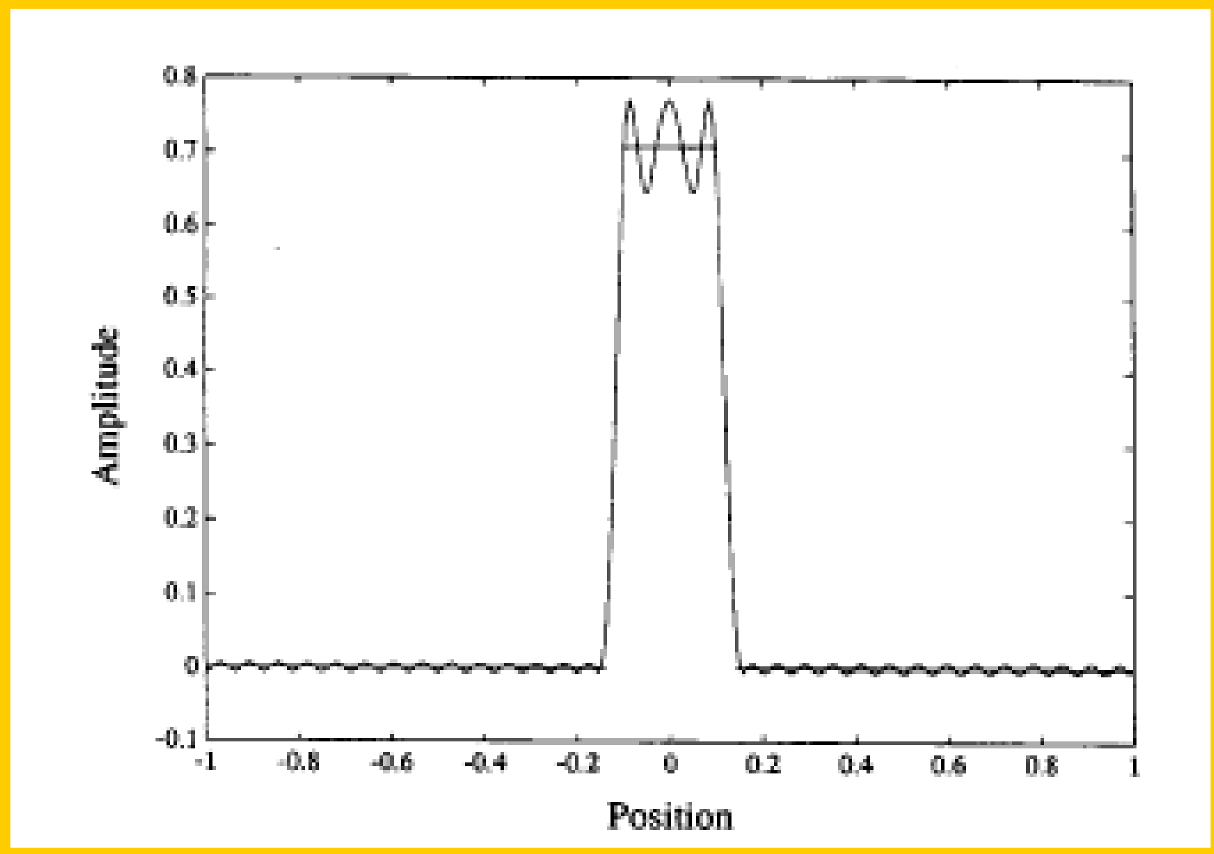
洪水猛獸終於遠離了

休息一下唄~

RF Design

- 如何由$A_n(z)$、$B_n(z)$得到Slice Profile？
  - $B_n(z)$ is used to optimally approximate the ideal slice profile.
  - $|A_n(z)|$ needs to be chosen consistent with $B_n(z)$ according to $|A_n(z)|^2 + |B_n(z)|^2 = 1$ and $A_n(z)$ is related to in-slice phase. (evaluated along the unit circle $e^{iGx\Delta t}$)

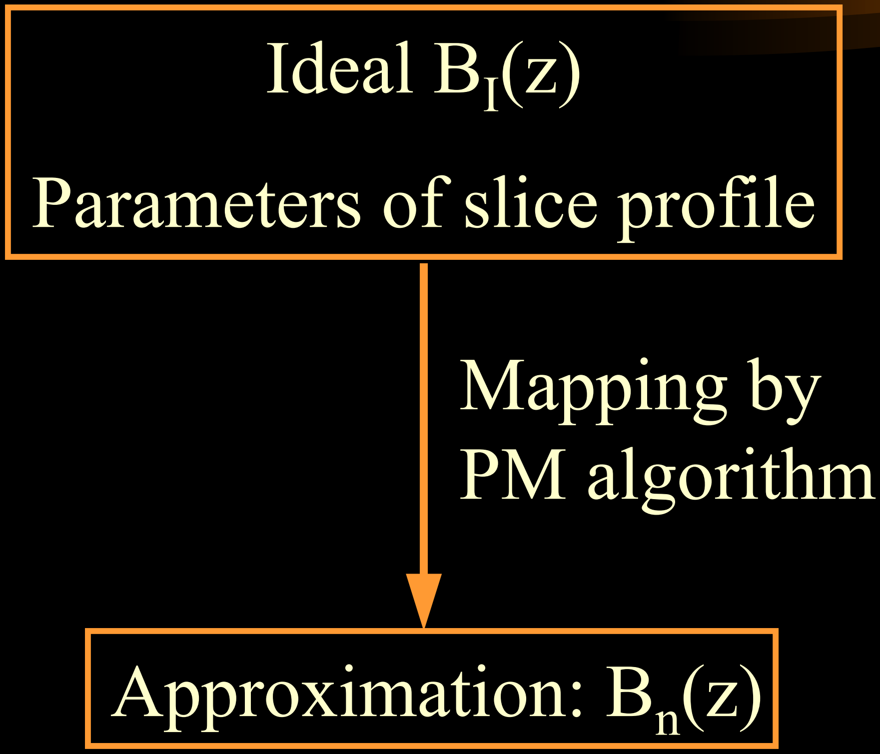
What we want &
What we'll really have



$B_n(z)$

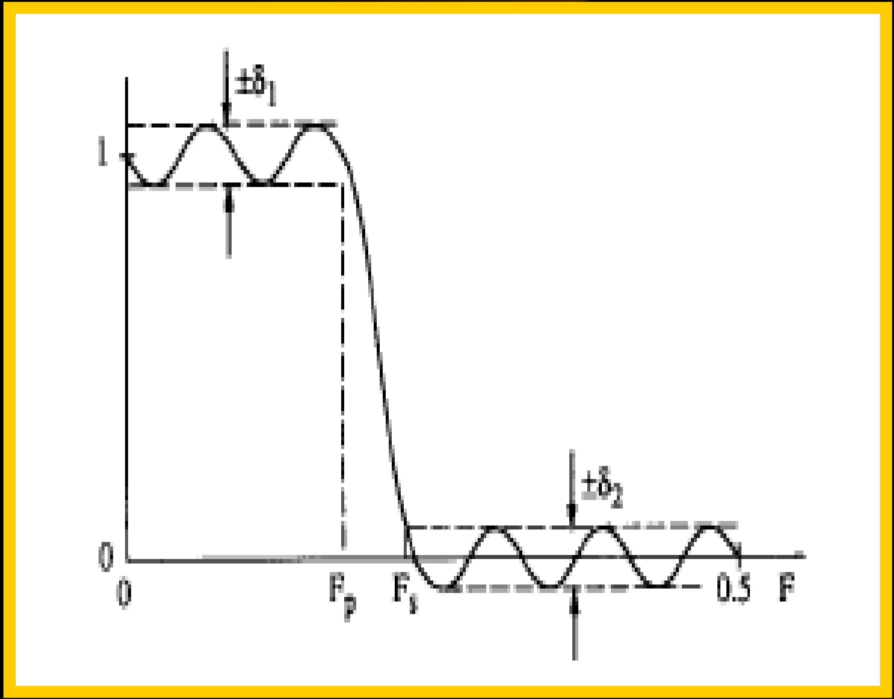
- $B_n(z)$ is used to optimally approximate the ideal slice profile.
  - $B_n(z) \propto \sin(\phi/2)$, ϕ is the flip angle at position x.
  - Parks-McClellan algorithm for the design of finite impulse response (FIR) digital filters

More about $B_n(z)$



Digital Filter Parameters

- In-slice ripple δ_1
- Out-of-slice ripple δ_2
- Transition width
- Pulse length $N\Delta t$
- Bandwidth

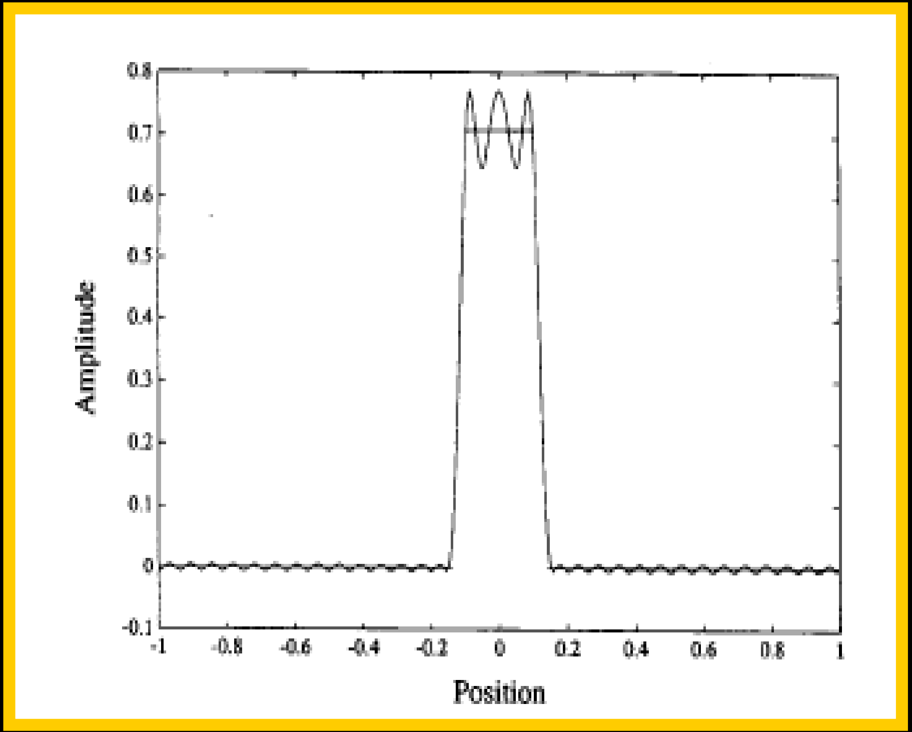


How about $A_n(z)$?

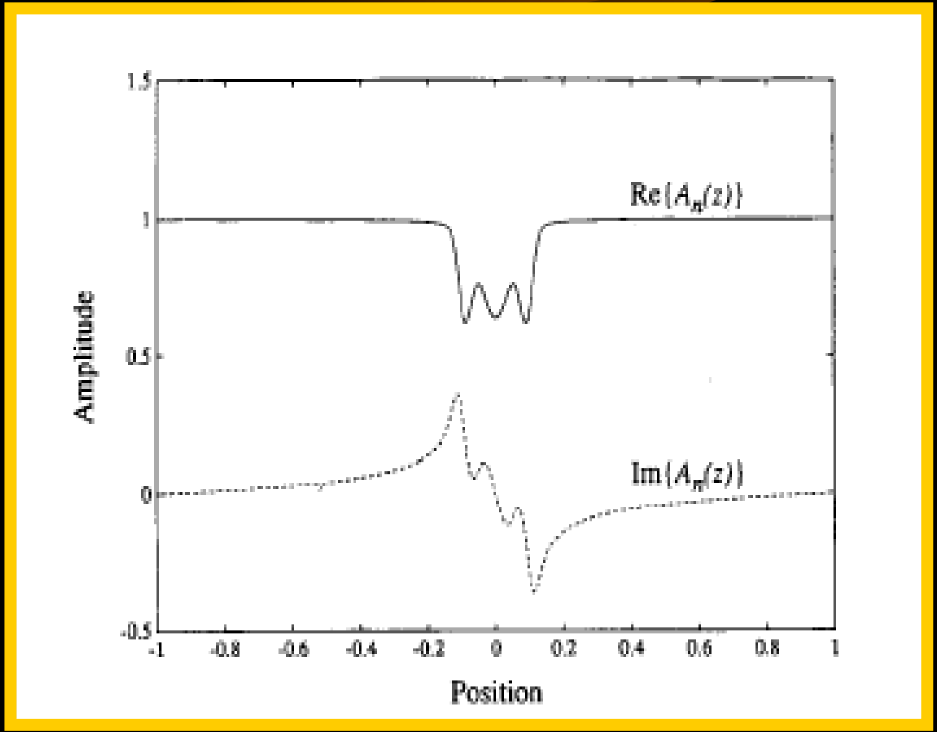
$$|A_n(z)| = \sqrt{1 - B_n(z) B_n^*(z)}$$

- $A_n(z)$ is related to in-slice phase, which is proportional to RF power.
  - Linear phase (spin echo pulse)
  - Minimum phase (when phase is not important)
  - Maximum phase (saturation, inversion)

$B_n(z)$ and $A_n(z)$

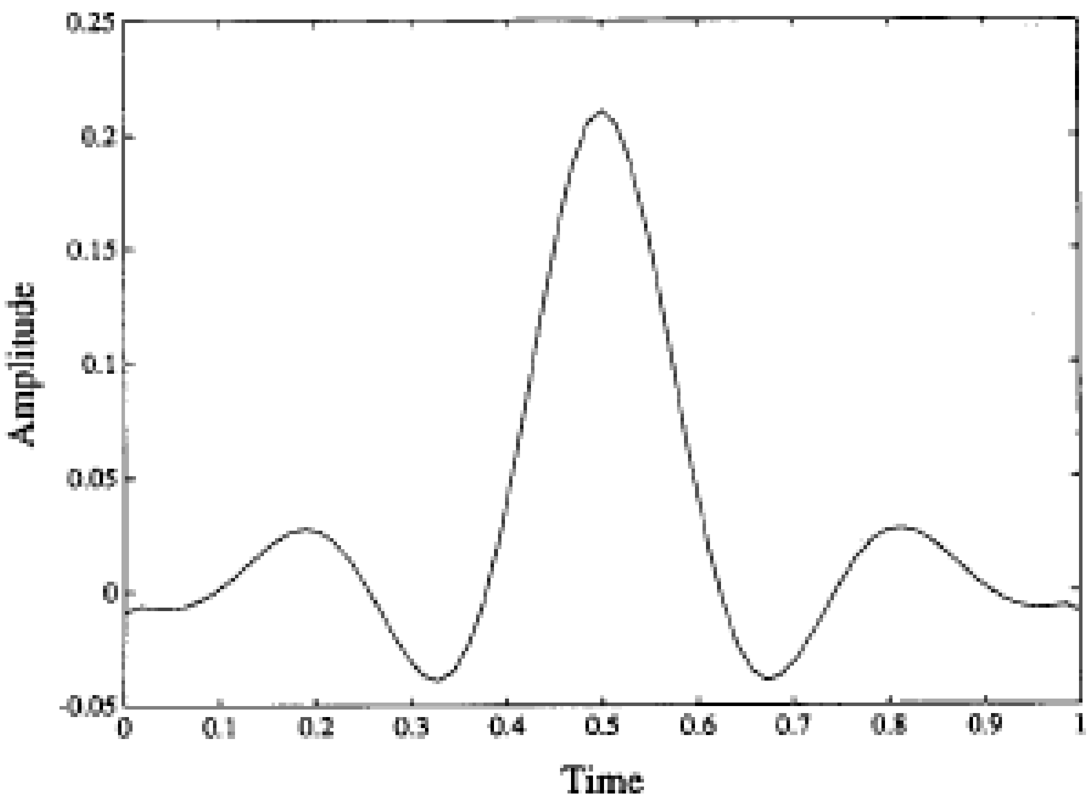


B_I and B_n



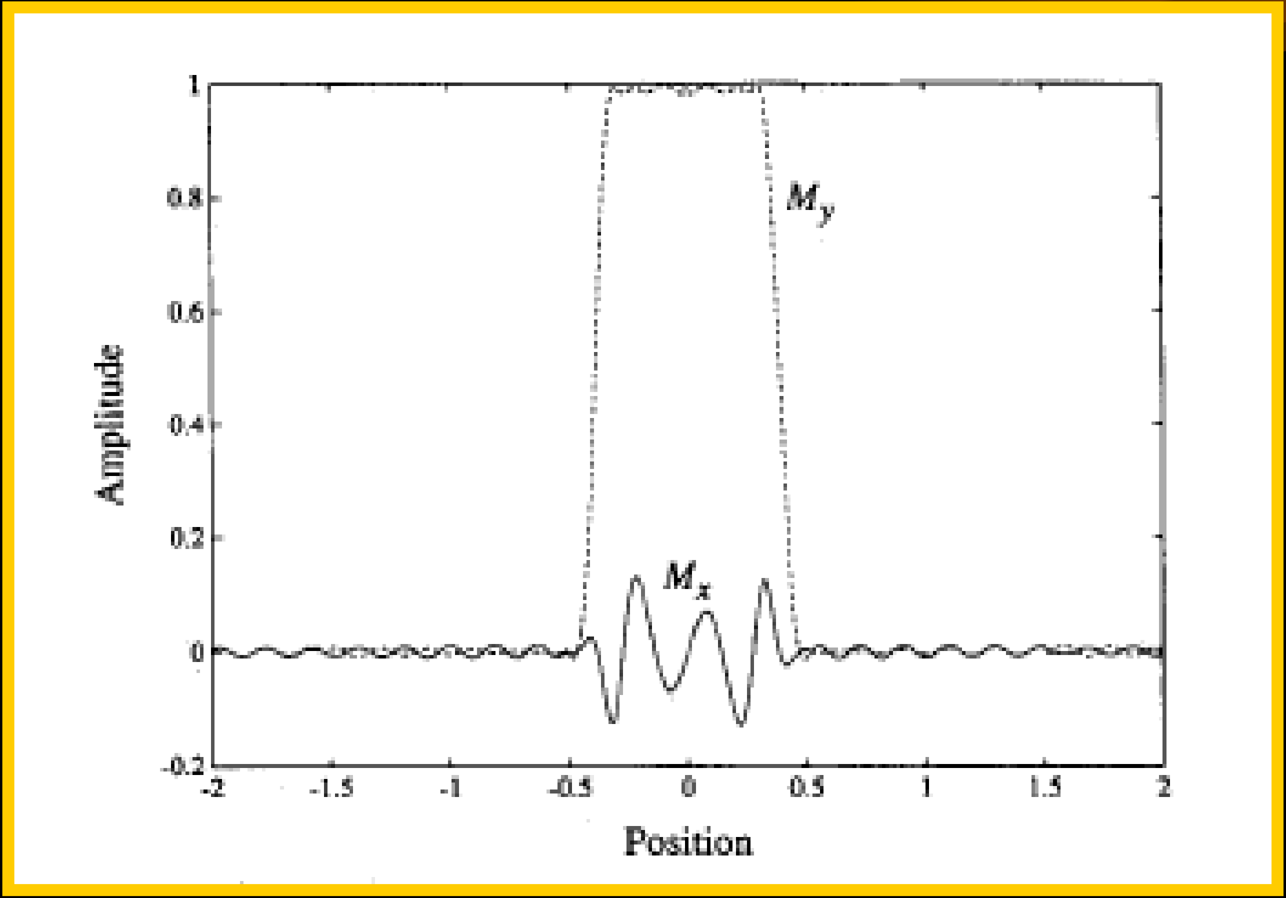
A_n with min. phase

SLR π/2 Pulse



Amplitude
0.25
0.2
0.15
0.1
0.05
0
-0.05
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Time

Slice Profile

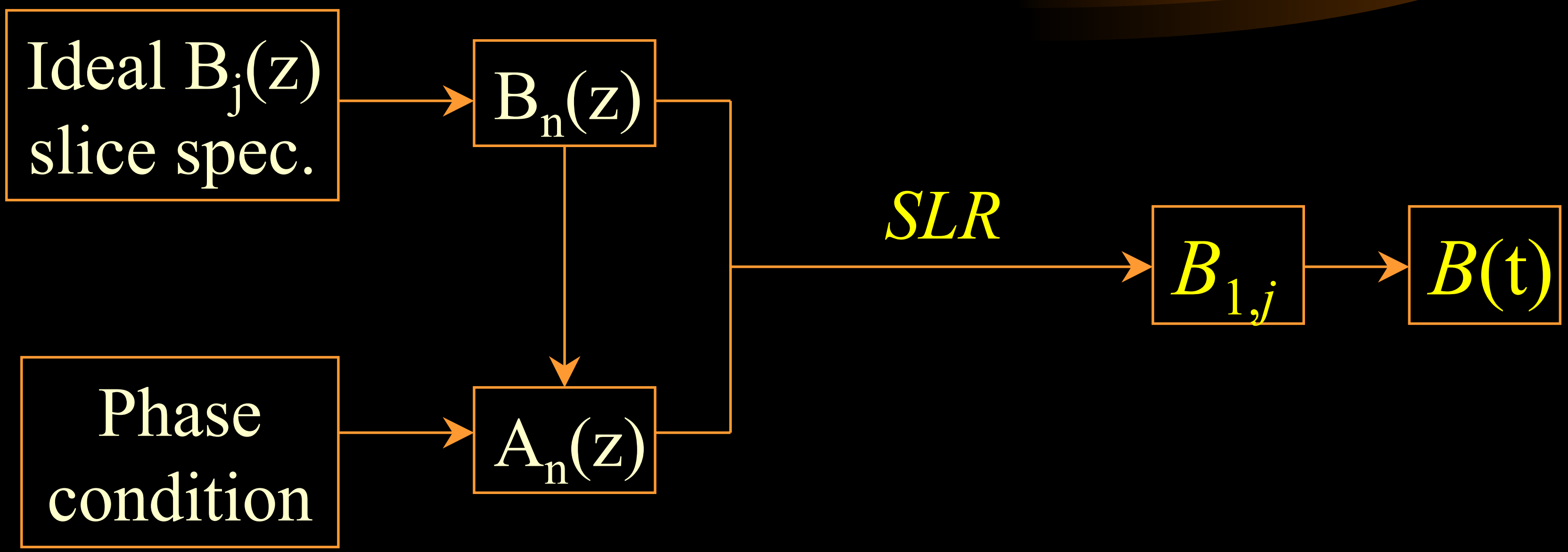


整理一下

- 我們所希望的

激發後磁矩的狀態 M_x、M_y → 射頻激發的波形 $B_{1,j}$, j=1…N

我們所能得到的

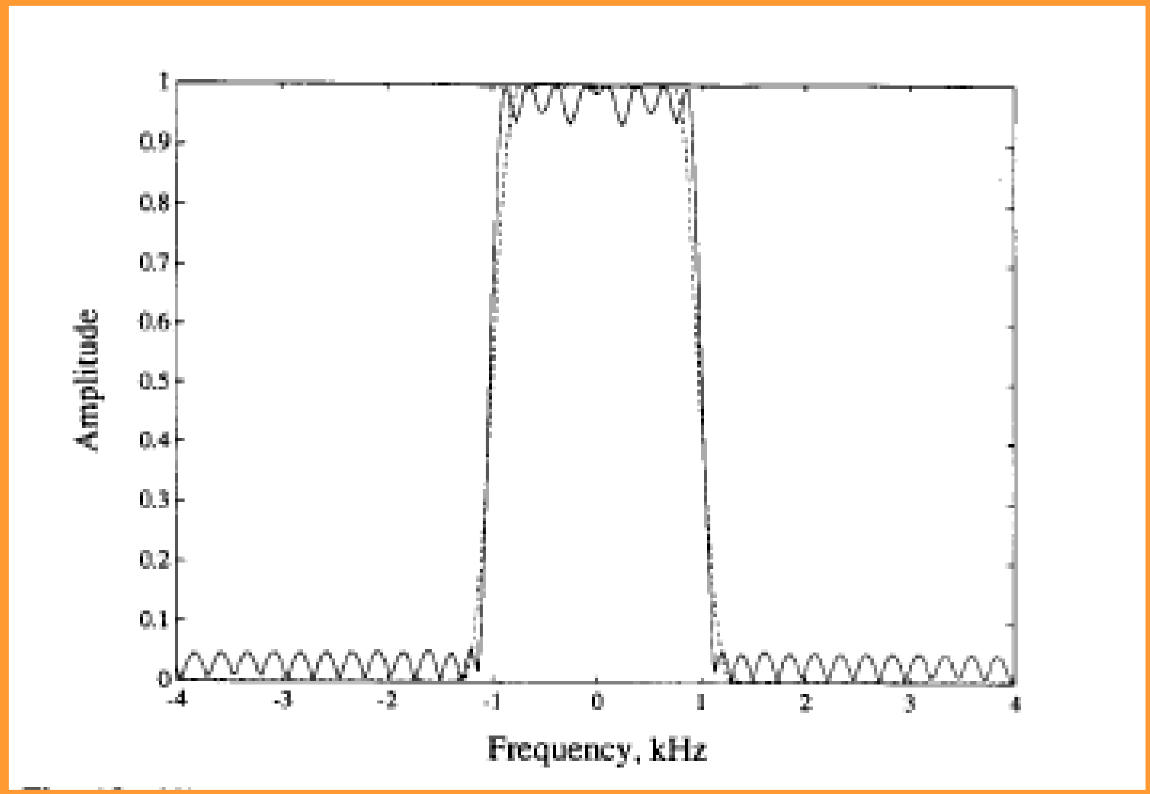


功德圓滿　阿彌陀佛

Trade-off between Profile Parameters

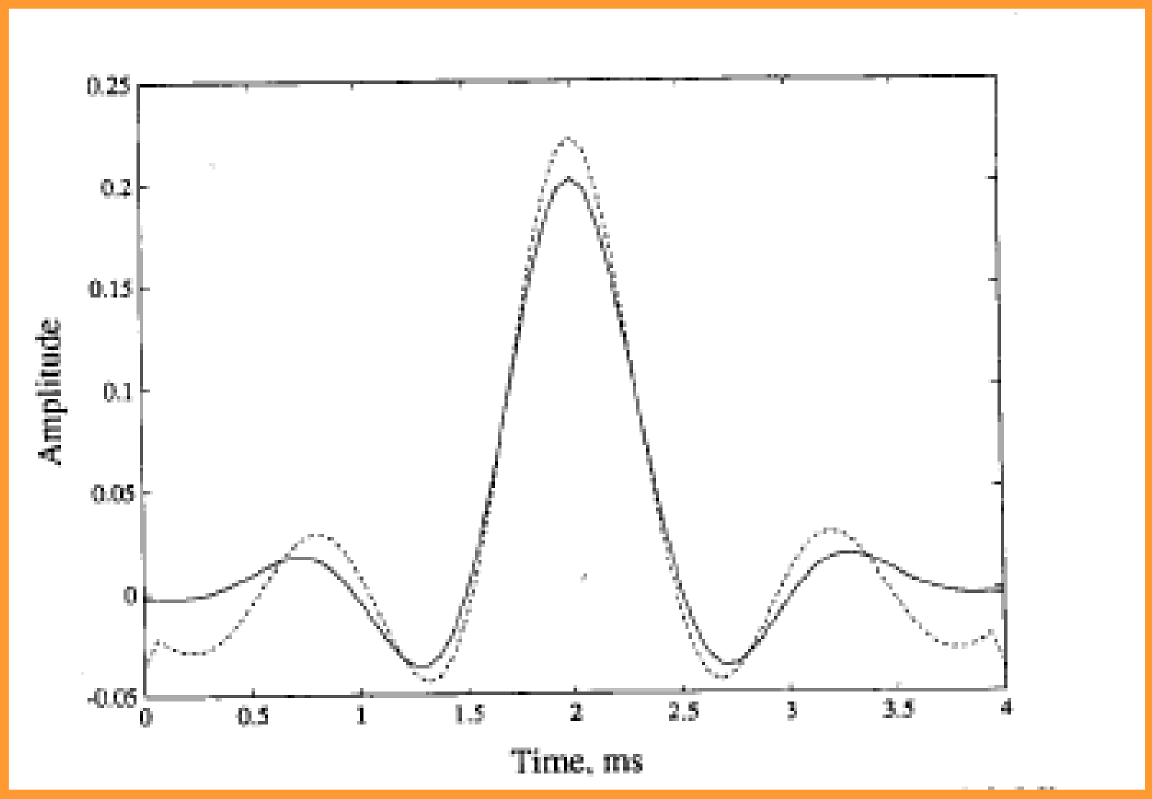
- Ripples, transition width, pulse length
  - The smaller, the better!
  - 有一好　沒兩好

Trade-off



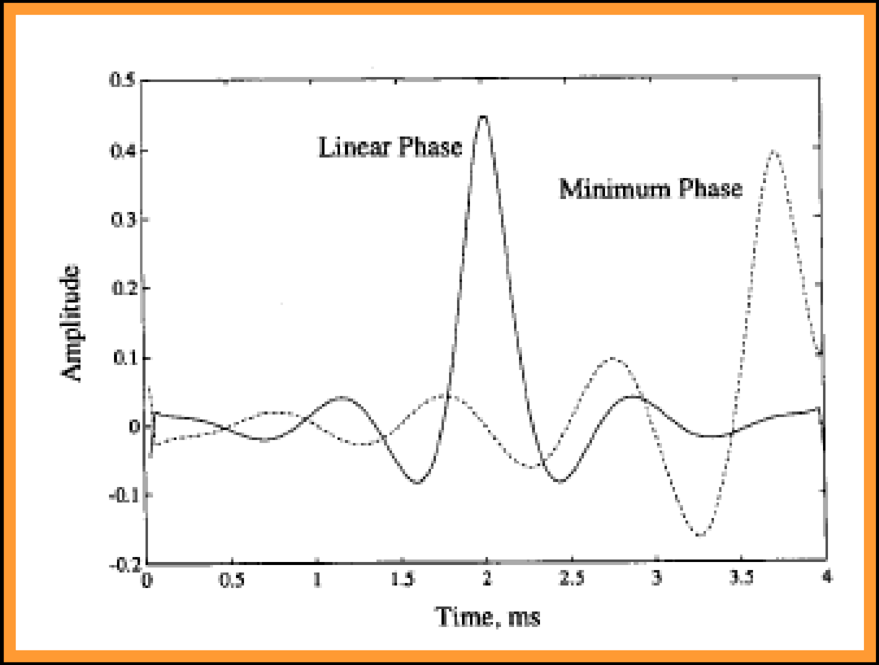
—Solid line: 5% ripple ---Dash line: 0.2% ripple

Trade-off

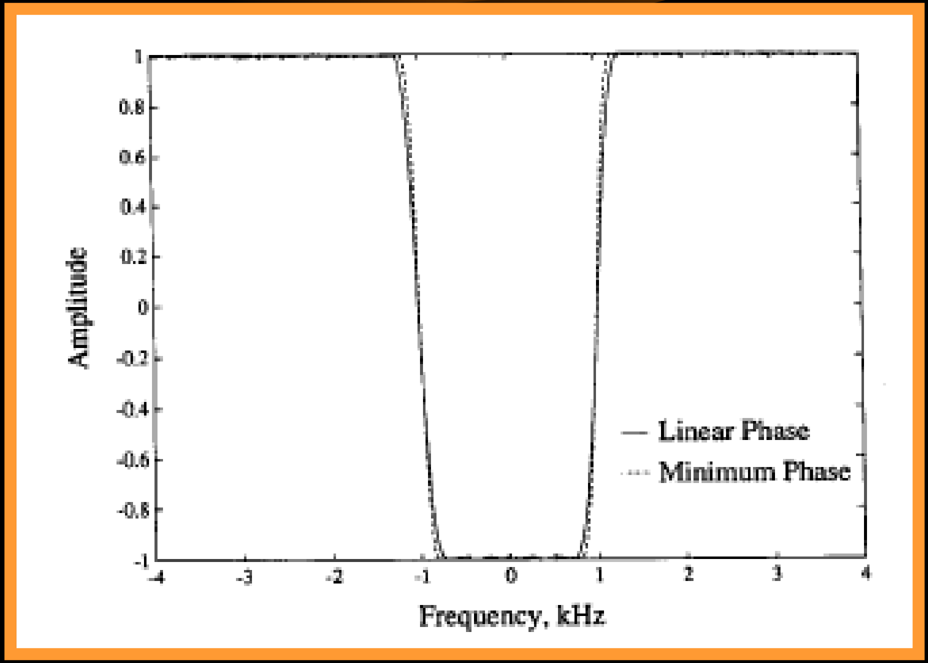


—Solid line: 5% ripple ---Dash line: 0.2% ripple

Trade-off

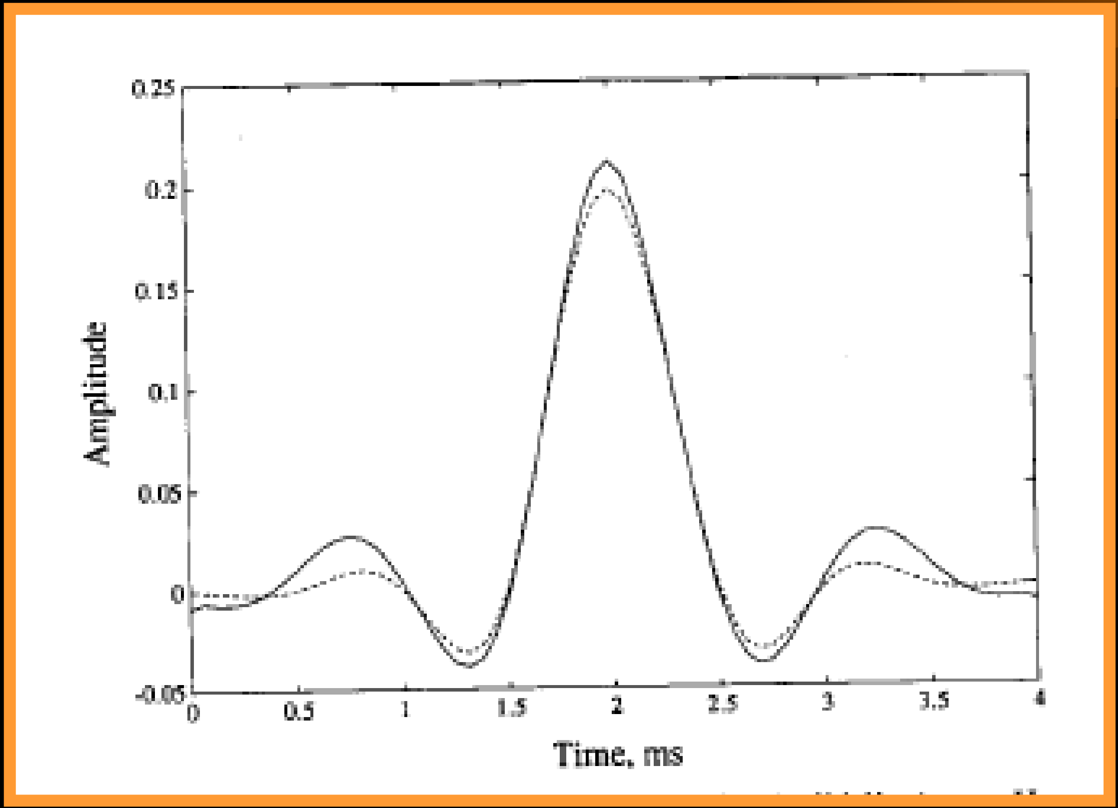


0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
Amplitude
Linear Phase
Minimum Phase
0
0.5
1
1.5
2
2.5
3
3.5
4
Time, ms



1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
Amplitude
Linear Phase
Minimum Phase
-4
-3
-2
-1
0
1
2
3
4
Frequency, kHz

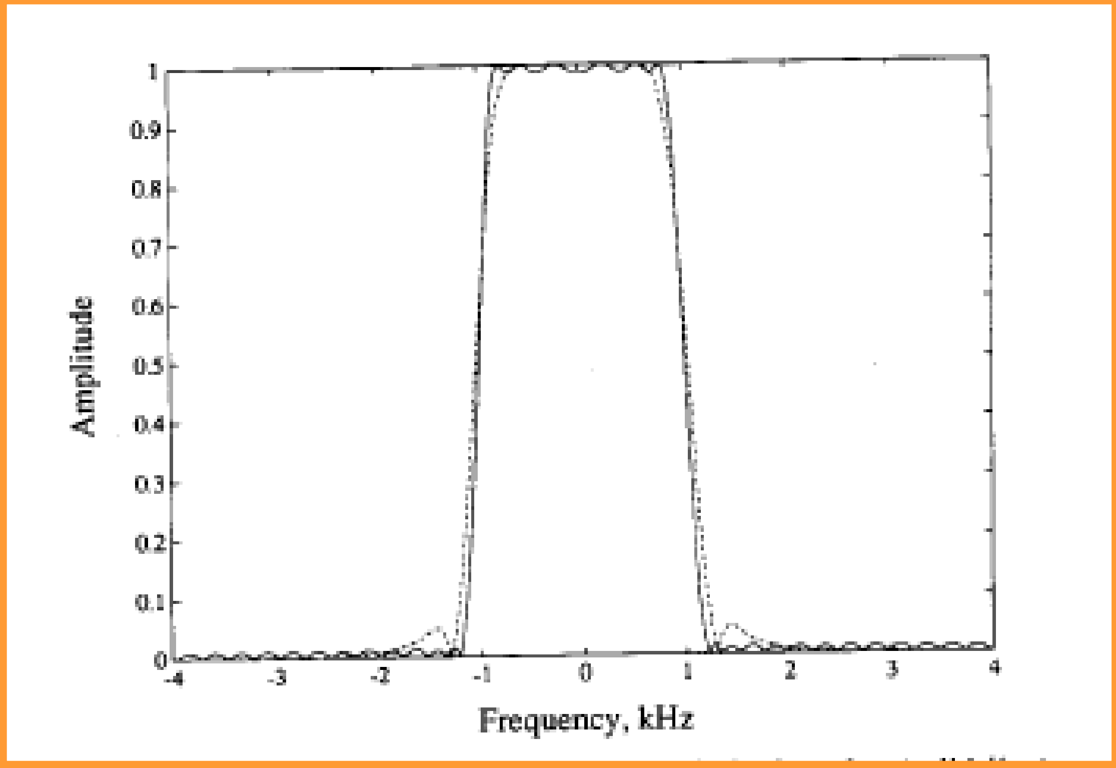
RF波形明明長得很像sinc



— solid line: SLR 1% ripple pulse

--- dash line: Hamming windowed sinc pulse

效果真的有差嗎？



— solid line: SLR 1% ripple pulse

--- dash line: Hamming windowed sinc pulse

Summary

$B_1(t)$ ⟷ $M(x,t)$

Forward SLR

Inverse SLR

$B_1(t)$ ⟷ Bloch Equation: Several assumptions

$M(x,t)$ ⟷ $A_n(z)$, $B_n(z)$: FIR filter mapping

Bloch Equation ⟺ $\begin{pmatrix} \alpha & -\beta^* \\ \beta & \alpha^* \end{pmatrix}$ ⟺ $A_n(z)$, $B_n(z)$

Summary

- Pulse sequences have not to be designed around available RF pulses.
- Slice profile parameters can be traded off analytically.

Reference

- [1] J. Pauly, P. Le Roux, D. Nishimura, and A. Macovski, "Parameter Relations for the Shinnar-Le Roux Selective Excitation Pulse Design Algorithm," IEEE Trans. Med. Imaging, vol. 10, p.53-65, March 1991

- [2] P.M. Joseph, L. Axel, M. O'Donnell, "Potential Problems with Selective Pulses in NMR Imaging Systems," Med. Phys. 11(6), p.772-p.777, Nov/Dec 1984

Thanks you all!!

Especially Boss and Coco!