Tractography via Diffusion Tensor MRI

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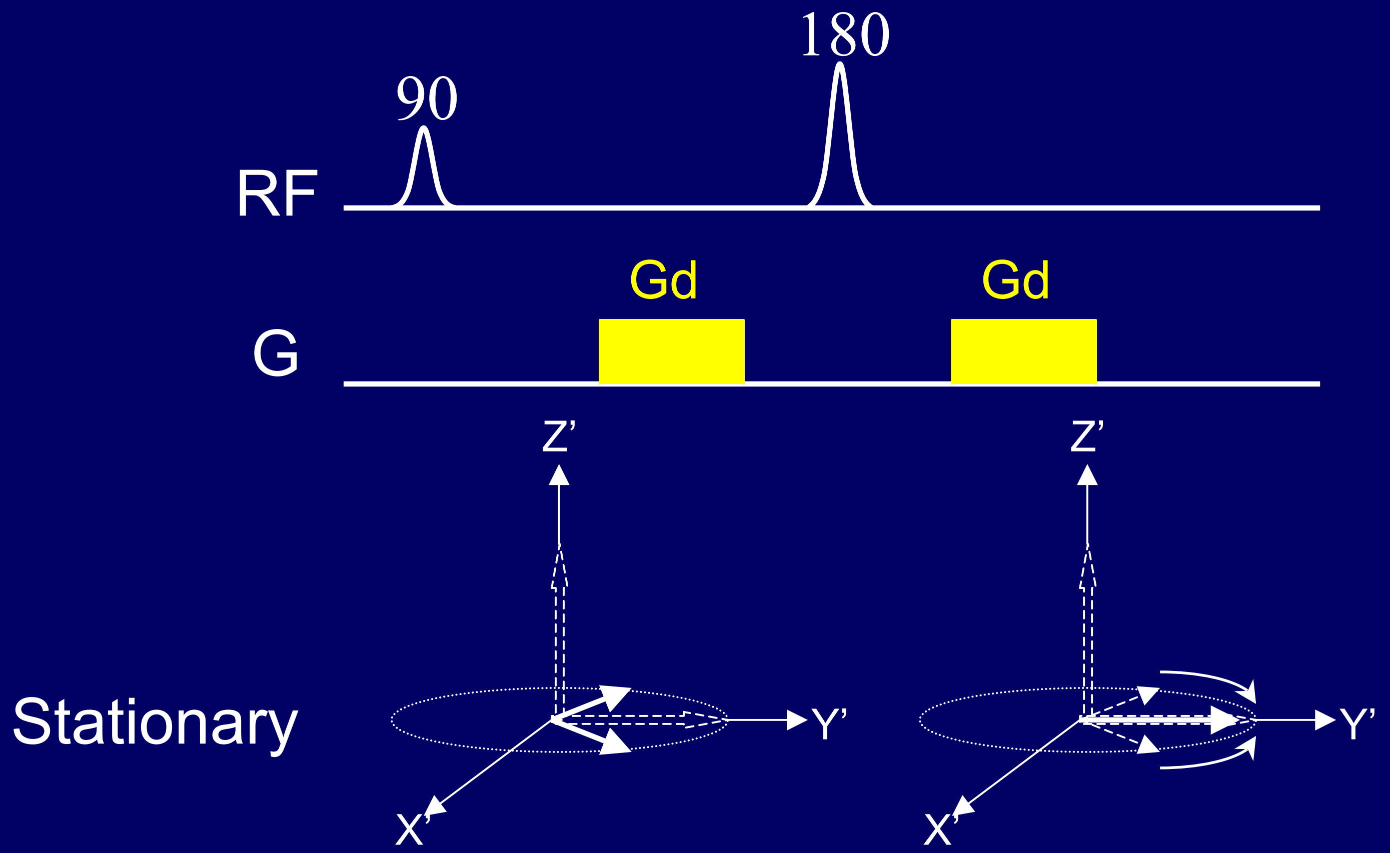
R91921041 周銘鐘

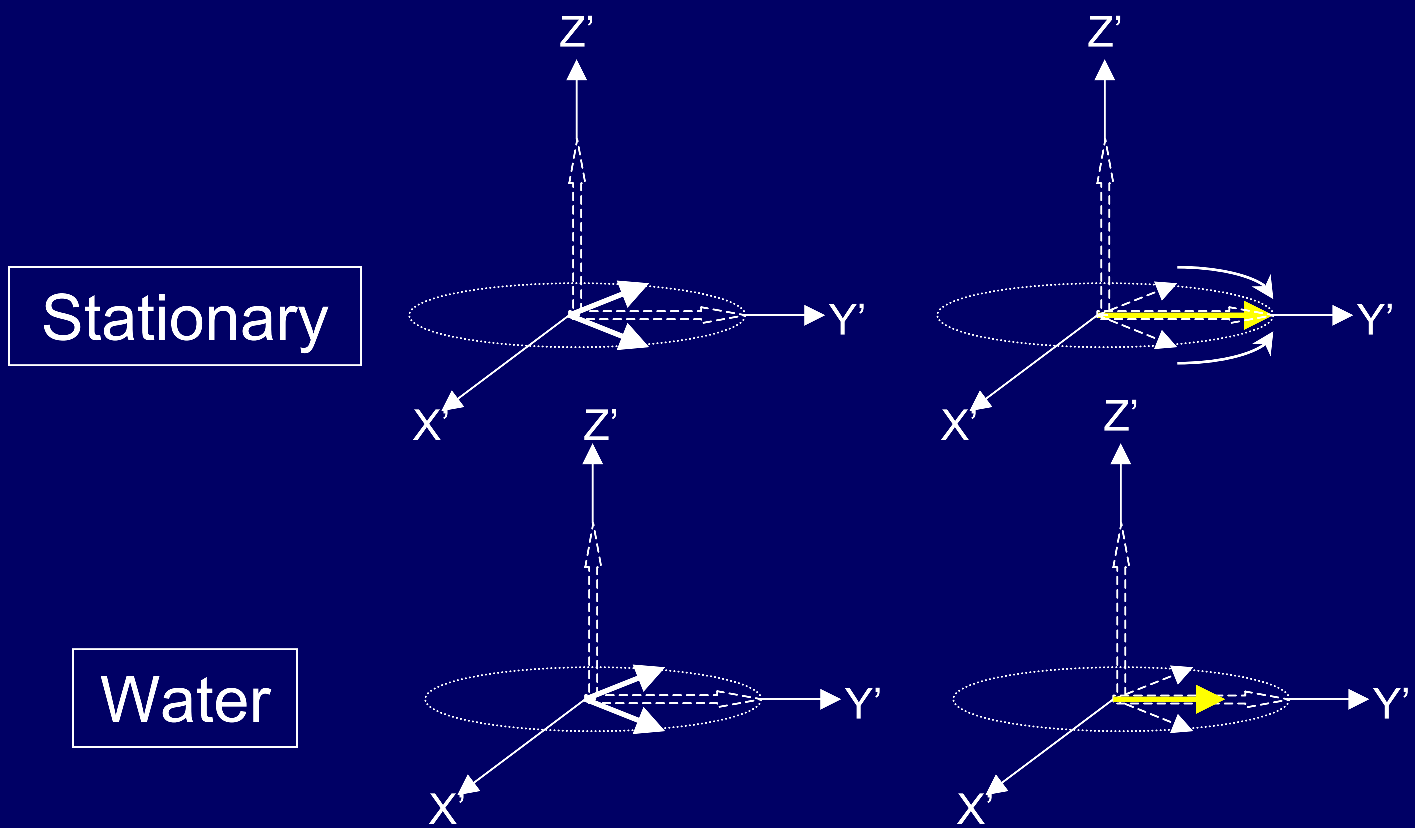
Diffusion Image

- Pulse sequence



Diffusion Image

- Rotating frame

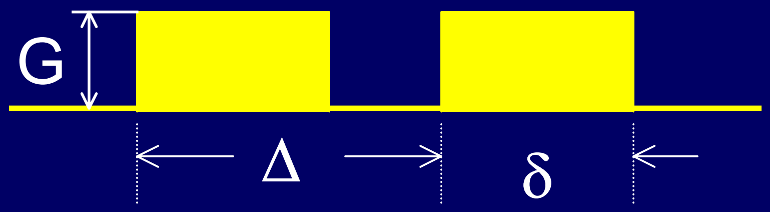


Diffusion Image

- DWI (Diffusion Weighted Image)

Diffusion Image

- Signal Decay
  - Gradient Strength & Duration
    - » $b = \gamma^2 G^2 \delta^2 (\Delta - \delta/3)$ ..roughly



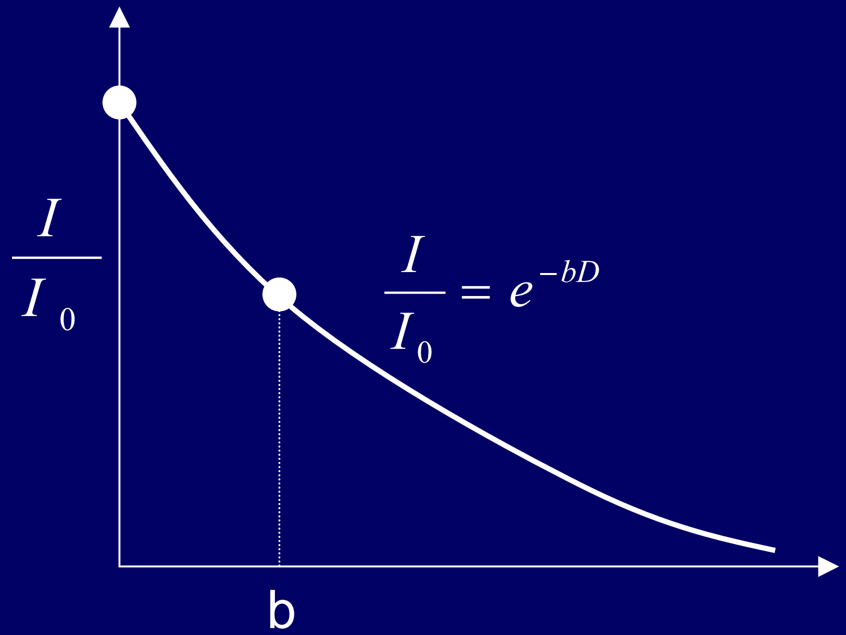
  - Diffusion Coefficient, D

Diffusion Image

- Signal Decay

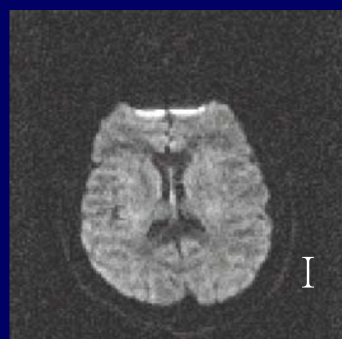
$$\ln\left(\frac{I}{I_0}\right) = -b \times D$$

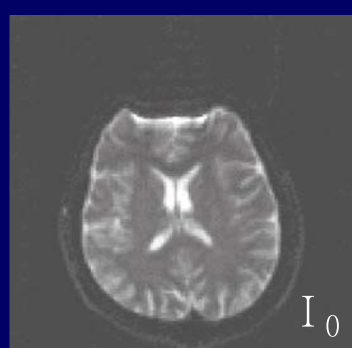
$$D = \frac{\ln\left(I / I_0\right)}{-b}$$



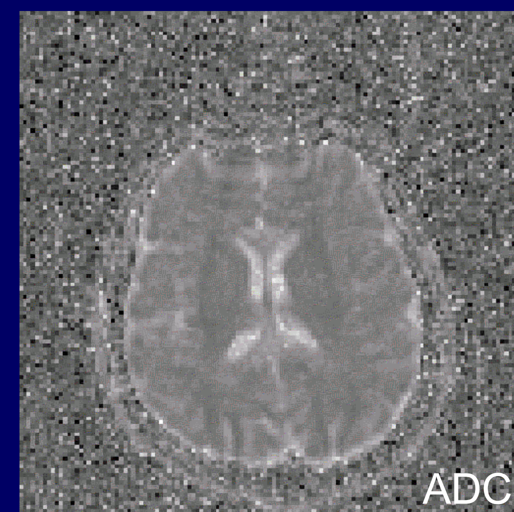
Diffusion Image

- ADC map (Apparent Diffusion Coefficient)





$$\ln\left(\frac{I}{I_0}\right) \times \frac{1}{-b} =$$

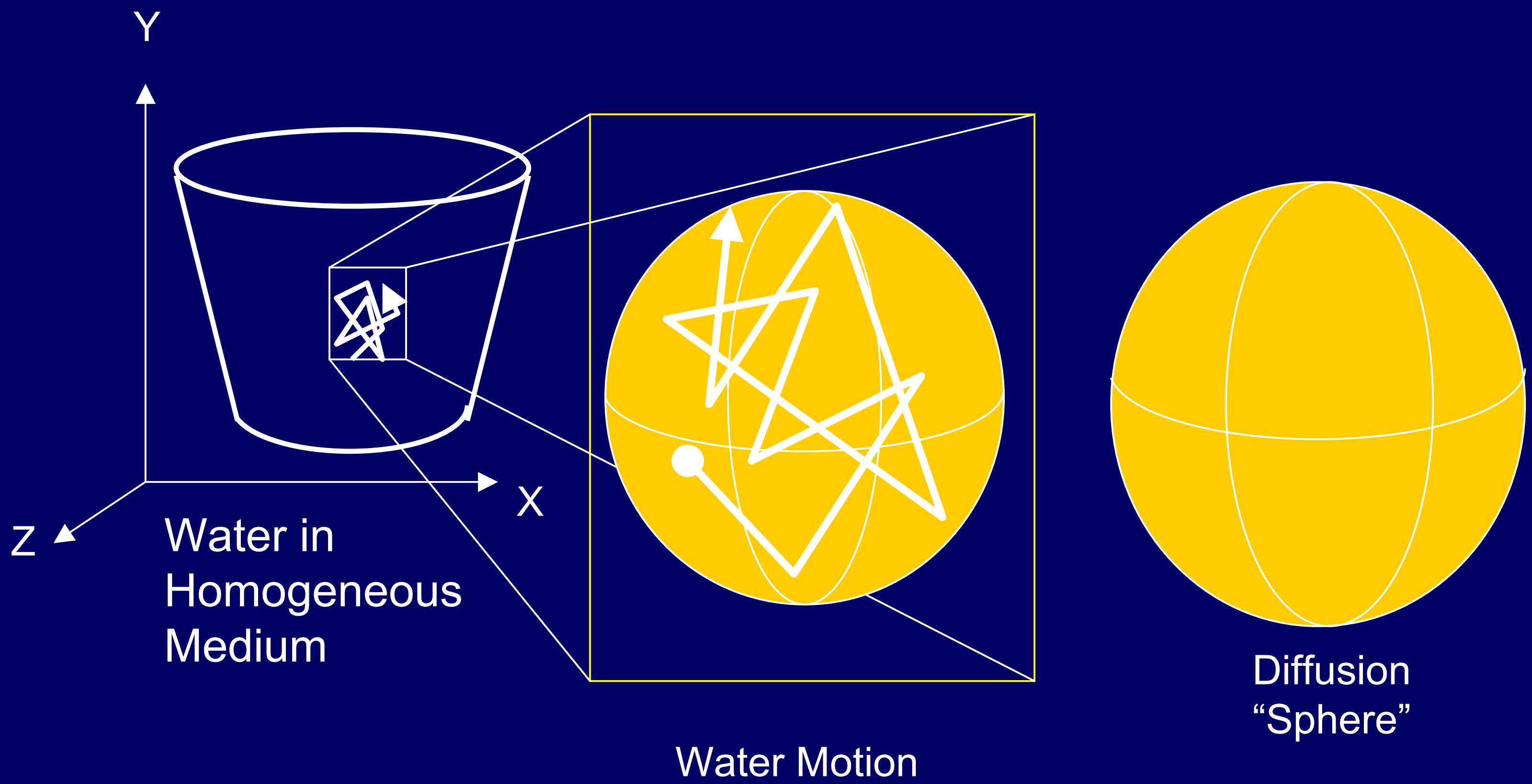


Diffusion Image

- Diffusion weighted image
  - Low intensity => high diffusivity
- ADC map
  - Signal intensity $\propto$ diffusivity
  - Measure in single direction

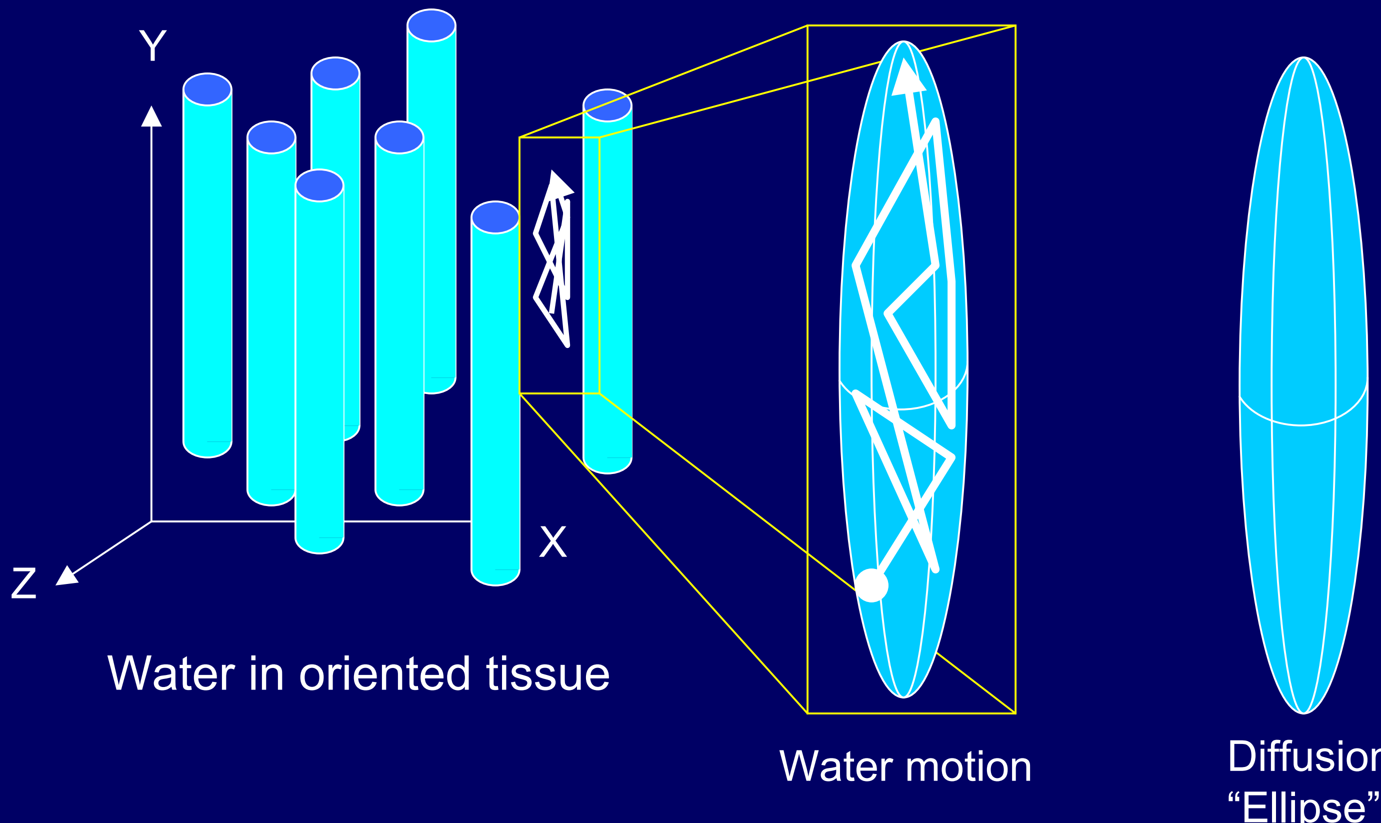
Diffusion Tensor MRI

- Isotropic Diffusion



Diffusion Tensor MRI

- Anisotropic Diffusion



Diffusion Tensor MRI

- Diffusion Ellipse
  - Ellipsoid Equation

$$D_{xx}x^2 + D_{yy}y^2 + D_{zz}z^2 + 2D_{xy}xy + 2D_{yz}yz + 2D_{xz}xz$$

$$= \begin{pmatrix} x & y & z \end{pmatrix} \times \begin{pmatrix} D_{xx} & D_{xy} & D_{xz} \\ D_{xy} & D_{yy} & D_{yz} \\ D_{xz} & D_{yz} & D_{zz} \end{pmatrix} \times \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

Diffusion Tensor MRI

- Tensor Matrix
  - G(1,1,0), G(-1,1,0), G(0,1,1), G(0,1,-1), G(1,0,1), G(1,0,-1)

$$\ln\left(\frac{I}{I_0}\right) = -bD = -\gamma^2\delta^2\left(\Delta - \frac{\delta}{3}\right)G^T D_{3\times3} G$$

$$\left(1\,1\,0\right)\times\begin{pmatrix} D_{xx} & D_{xy} & D_{xz} \\ D_{xy} & D_{yy} & D_{yz} \\ D_{xz} & D_{yz} & D_{zz} \end{pmatrix}\times\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = D_{xx} + D_{yy} + 2D_{xy}$$

Cross Term

Diffusion Tensor MRI

- Tensor Matrix

$$D_{(1,\,1,\,0)} = D_{xx} + D_{yy} + 2D_{xy}$$

$$D_{(1,-1,0)} = D_{xx} + D_{yy} - 2D_{xy}$$

$$D_{(1,\,0,\,1)} = D_{xx} + D_{zz} + 2D_{xz}$$

$$D_{(1,0,-1)} = D_{xx} + D_{zz} - 2D_{xz}$$

$$D_{(0,\,1,\,1)} = D_{yy} + D_{zz} + 2D_{yz}$$

$$D_{(0,1,-1)} = D_{yy} + D_{zz} - 2D_{yz}$$

Diffusion Tensor MRI

- Tensor Matrix

$$\begin{pmatrix} D_{(1,\ 1,0)} \\ D_{(1,-1,0)} \\ D_{(1,\ 0,1)} \\ D_{(1,0,-1)} \\ D_{(0,\ 1,1)} \\ D_{(0,1,-1)} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 2 & 0 & 0 \\ 1 & 1 & 0 & -2 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 2 \\ 1 & 0 & 1 & 0 & 0 & -2 \\ 0 & 1 & 1 & 0 & 2 & 0 \\ 0 & 1 & 1 & 0 & -2 & 0 \end{pmatrix} \times \begin{pmatrix} D_{xx} \\ D_{yy} \\ D_{zz} \\ D_{xy} \\ D_{yz} \\ D_{xz} \end{pmatrix}$$

Diffusion Tensor MRI

- Tensor Matrix

$$\begin{pmatrix} D_{xx} \\ D_{yy} \\ D_{zz} \\ D_{xy} \\ D_{yz} \\ D_{xz} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 2 & 0 & 0 \\ 1 & 1 & 0 & -2 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 2 \\ 1 & 0 & 1 & 0 & 0 & -2 \\ 0 & 1 & 1 & 0 & 2 & 0 \\ 0 & 1 & 1 & 0 & -2 & 0 \end{pmatrix}^{-1} \times \begin{pmatrix} D_{(1,\ 1,0)} \\ D_{(1,-1,0)} \\ D_{(1,\ 0,1)} \\ D_{(1,0,-1)} \\ D_{(0,\ 1,1)} \\ D_{(0,1,-1)} \end{pmatrix}$$

Diffusion Tensor MRI

- Tensor Matrix

$$\vec{D}_{3x3} = \begin{pmatrix} D_{xx} & D_{xy} & D_{xz} \\ D_{xy} & D_{yy} & D_{yz} \\ D_{xz} & D_{yz} & D_{zz} \end{pmatrix}$$

Diffusion Tensor MRI

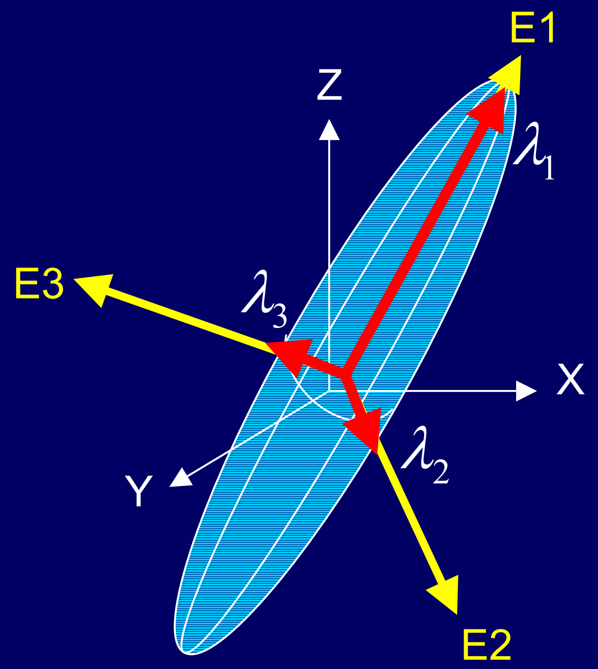
- Diagonalization

$$E^T \vec{D} E = \lambda_{3\times 3}$$

E = Eigen Vector

λ = Eigen Value

$\vec{D}$ = Tensor Matrix



$$\lambda_{3\times 3} = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}$$

Diffusion Tensor MRI

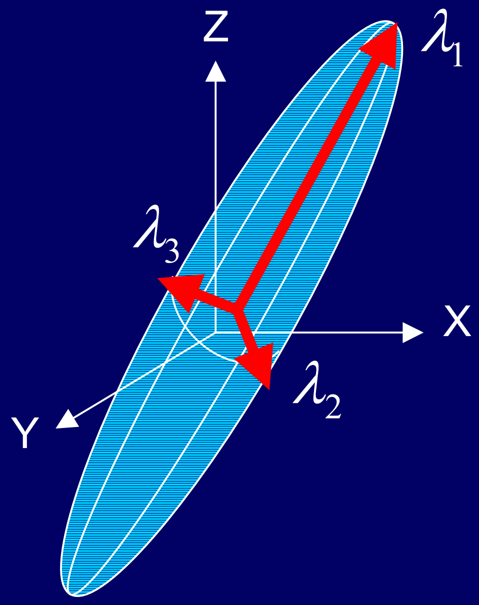
- Diffusivity Index
  - TraceADC
- Diffusion Anisotropy Indices
  - Volume Ratio (VR)
  - Relative Anisotropy (RA)
  - Fractional Anisotropy (FA)

Diffusivity Index

- TraceADC

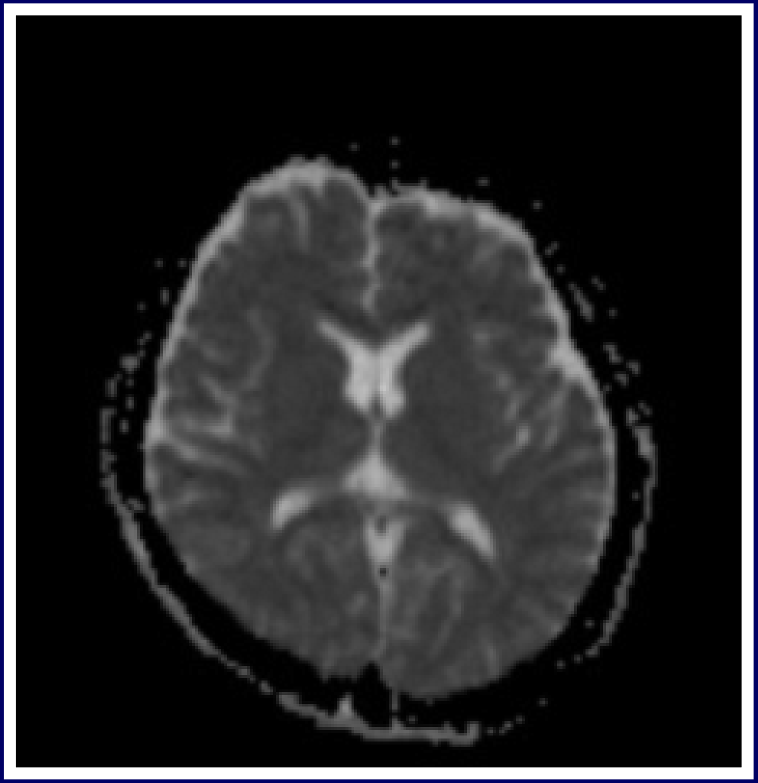
$$Trace\ ADC = \lambda_1 + \lambda_2 + \lambda_3$$

TraceADC $\propto$ Diffusivity



Diffusivity Index

- TraceADC



Anisotropy Index

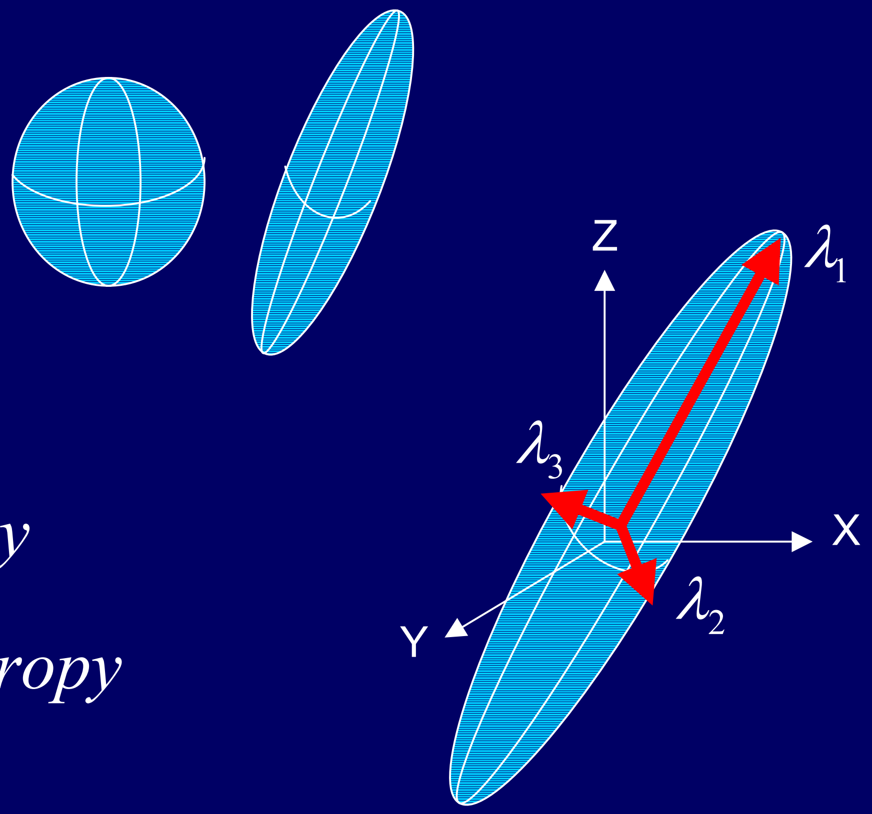
- Volume Ratio (VR)

$$VR = \frac{\lambda_1 \lambda_2 \lambda_3}{\bar{\lambda}^3}$$

$$\bar{\lambda} = \frac{\lambda_1 + \lambda_2 + \lambda_3}{3}$$

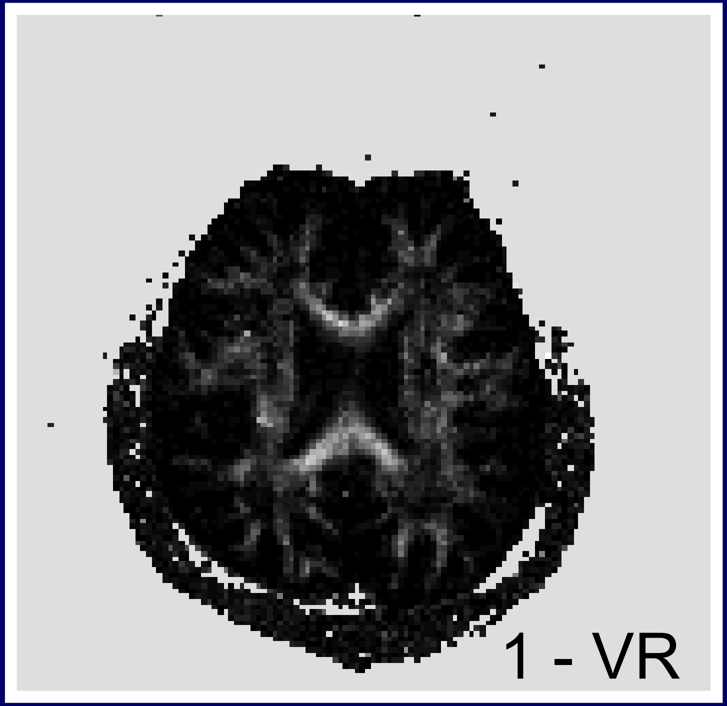
$$VR_{\max} = 1 = Max\ Isotropy$$

$$VR_{\min} = 0 = Max\ Anisotropy$$



Anisotropy Index

- Volume Ratio (VR)

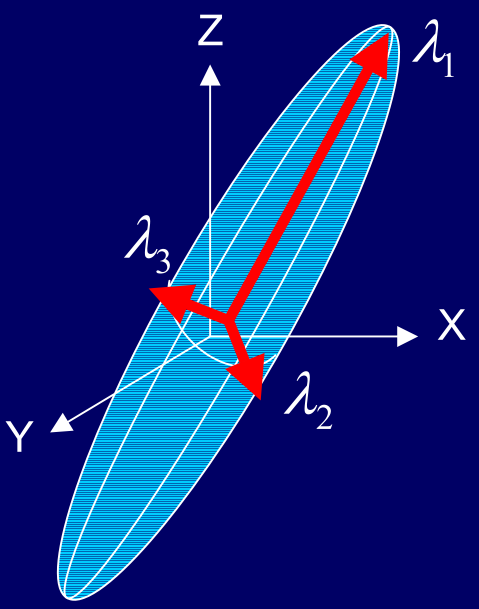


Anisotropy Index

- Relative Anisotropy (RA)

$$RA = \sqrt{(\lambda_1 - \bar{\lambda})^2 + (\lambda_2 - \bar{\lambda})^2 + (\lambda_2 - \bar{\lambda})^2} / \sqrt{3}\bar{\lambda}$$

$$\bar{\lambda} = \frac{\lambda_1 + \lambda_2 + \lambda_3}{3}$$

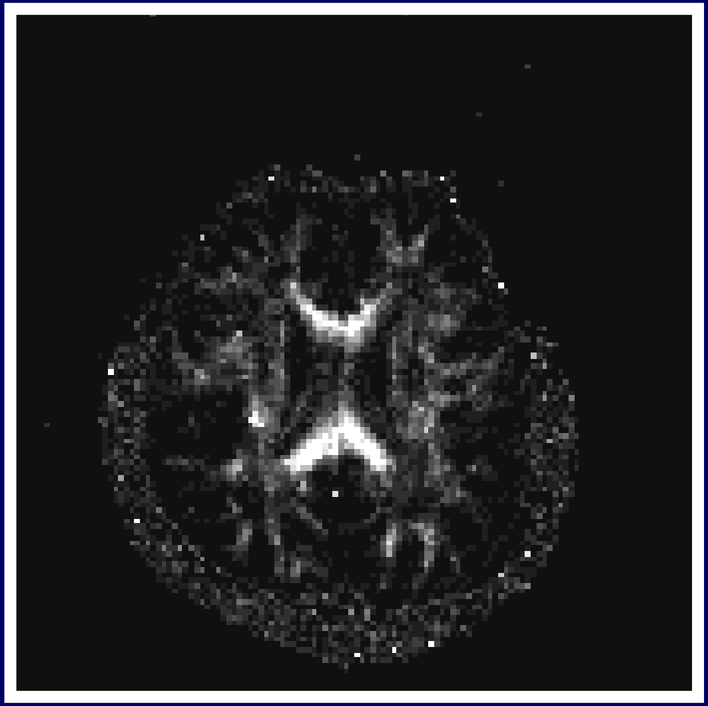


$$RA_{\max} = \frac{2}{\sqrt{3}} = Max\ Anisotropy$$

$$RA_{\min} = 0 = Max\ Isotropy$$

Anisotropy Index

- Relative Anisotropy (RA)



Anisotropy Index

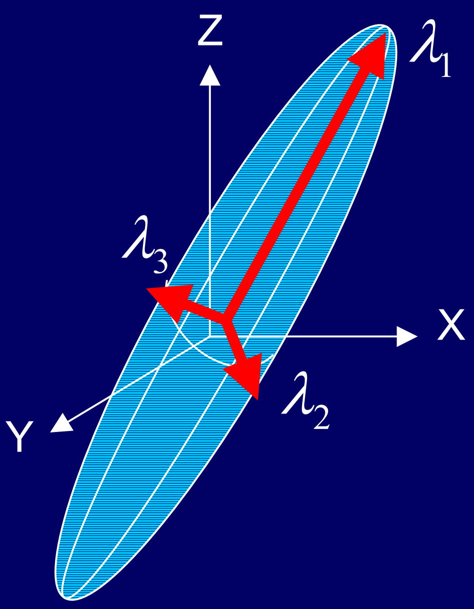
- Fractional Anisotropy (FA)

$$FA = \sqrt{\frac{3}{2}}\,\frac{\sqrt{(\lambda_1 - \bar{\lambda})^2 + (\lambda_2 - \bar{\lambda})^2 + (\lambda_2 - \bar{\lambda})^2}}{\sqrt{\lambda_1^{\,2} + \lambda_2^{\,2} + \lambda_3^{\,2}}}$$

$$\bar{\lambda} = \frac{\lambda_1 + \lambda_2 + \lambda_3}{3}$$

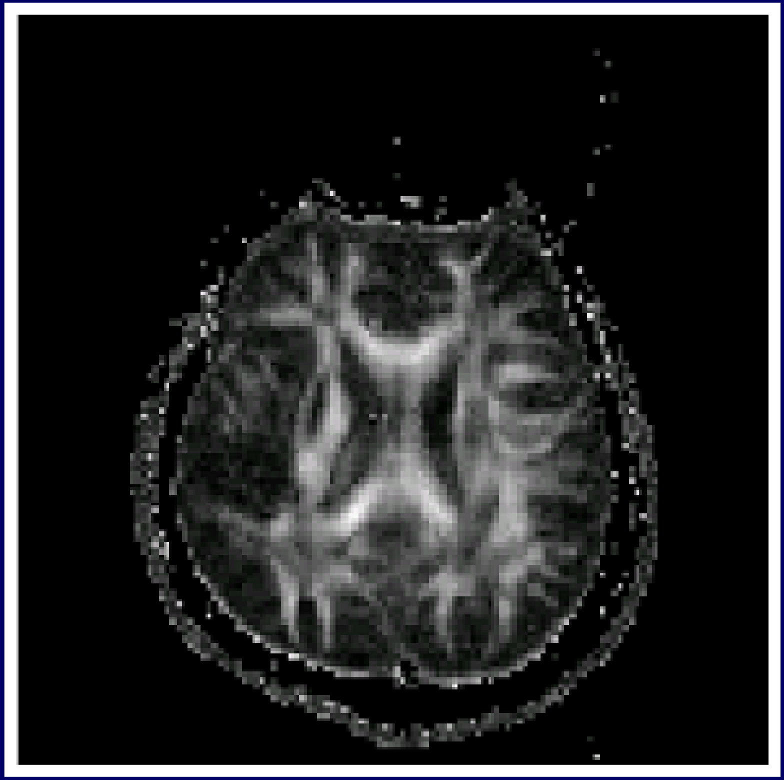
$$FA_{\max} = 1 = Max\ Anisotropy$$

$$FA_{\min} = 0 = Max\ Isotropy$$



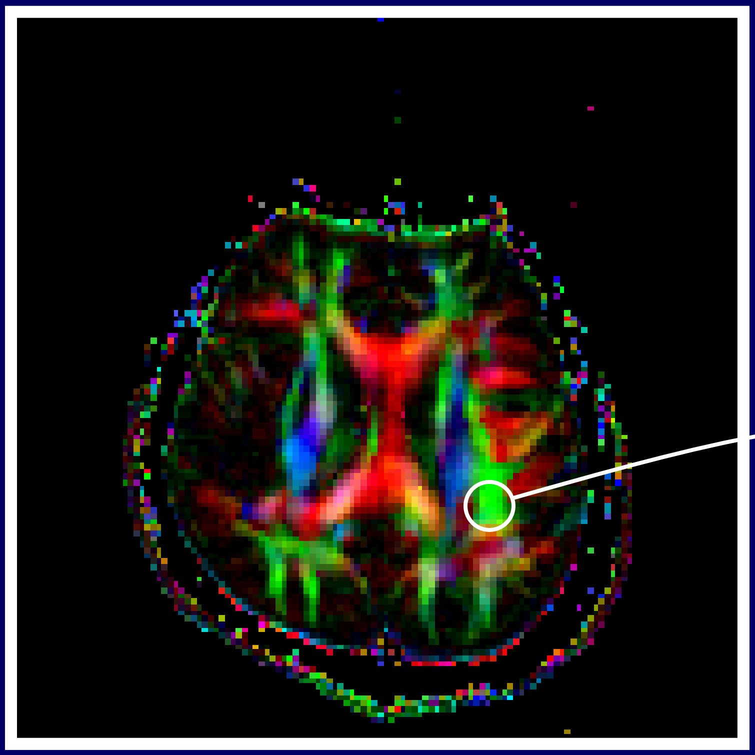
Anisotropy Index

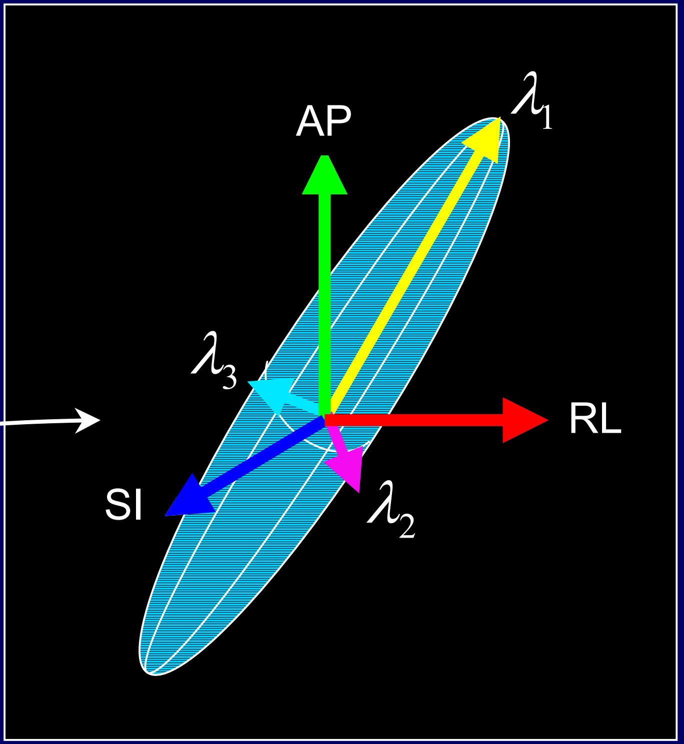
- Fractional Anisotropy (FA)



Diffusion Anisotropy

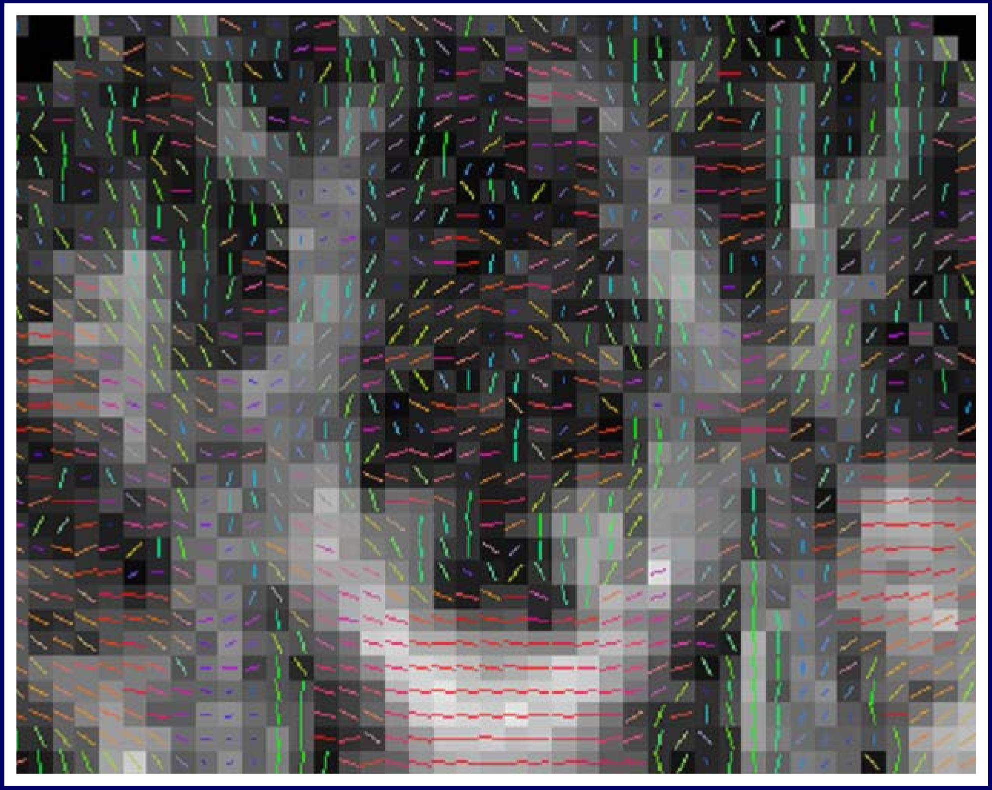
- Color-coded FA (axial view)

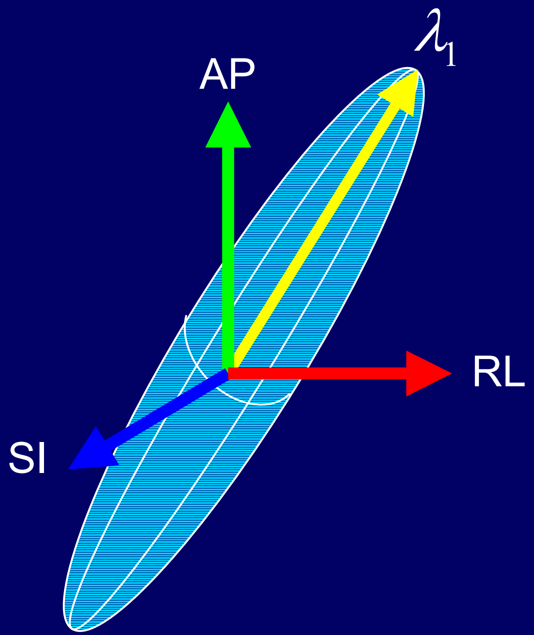




Diffusion Anisotropy

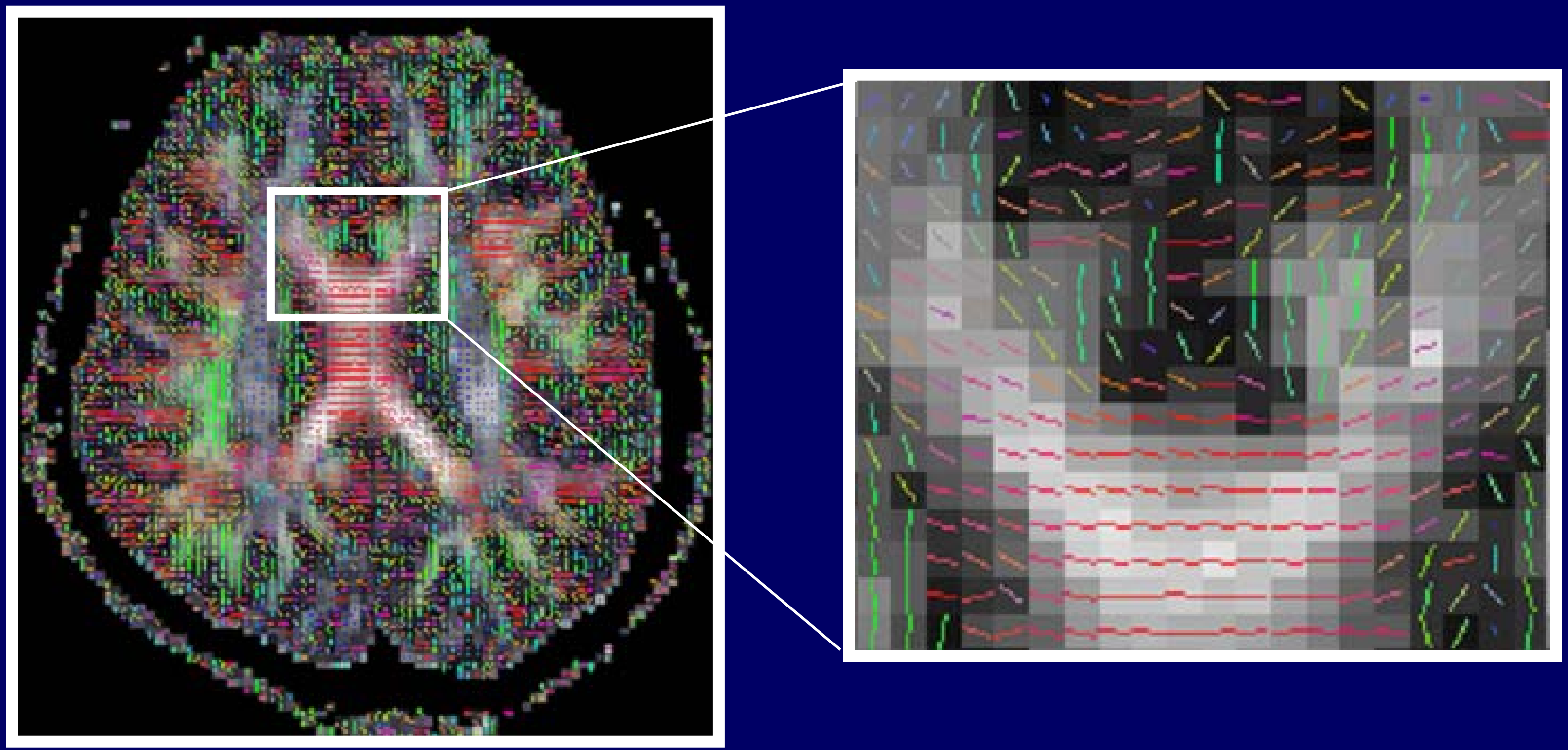
- Vector map FA (axial view)





Tractography

- Direction-coded FA (axial view)

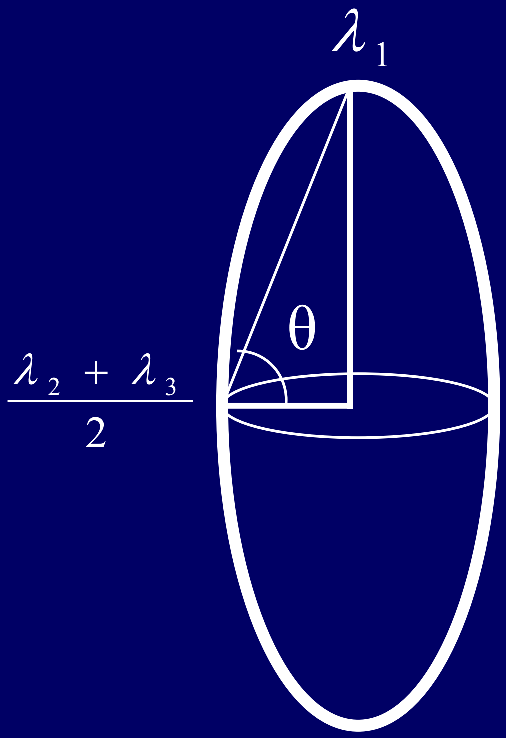


Tractography

- Discrete approach
  - EZ-Tracing
  - Terajima et al, JNR 2002
- Continuous approach
  - Fiber Tractography using DT-MRI data
  - Basser et al, MRM 2000

Discrete Approach

- Voxel filtering



$$\tan\theta = \frac{\lambda_1}{(\lambda_2 + \lambda_3)/2} = \frac{2\lambda_1}{\lambda_2 + \lambda_3}$$

θ : anisotropic angle

$$\tan\theta \geq 1.2 \approx FA \geq 0.1$$

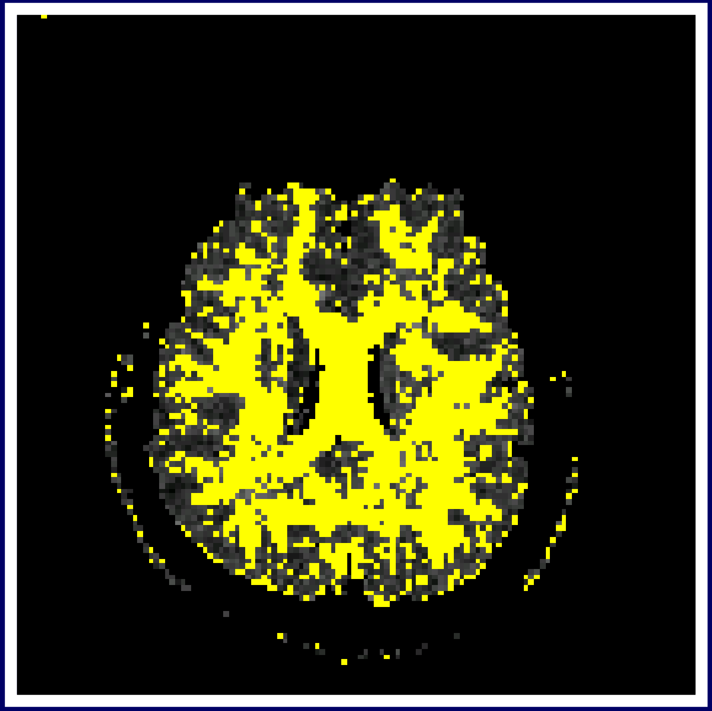
$$Trace \leq 0.003\,(mm^2/s)$$

Discrete Approach

- Voxel filtering

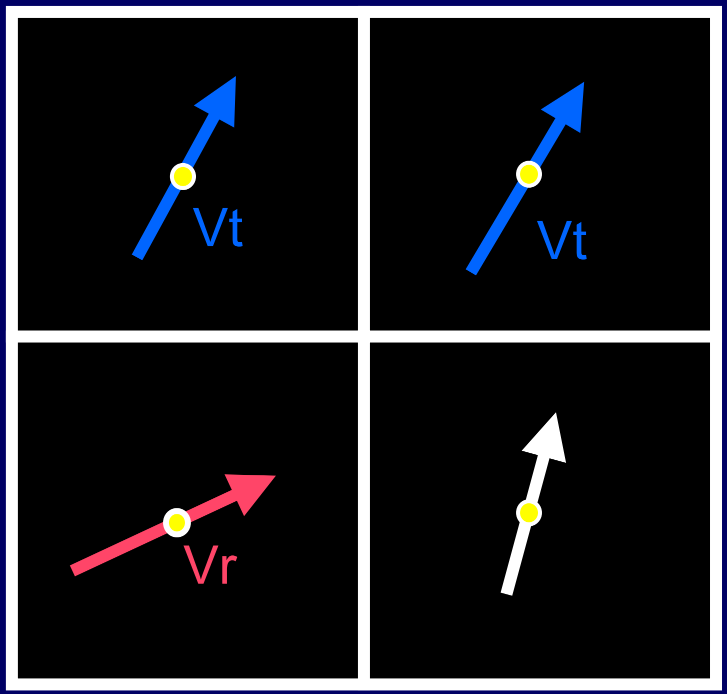
$$\tan\theta \geq 1.2 \approx FA \geq 0.1$$

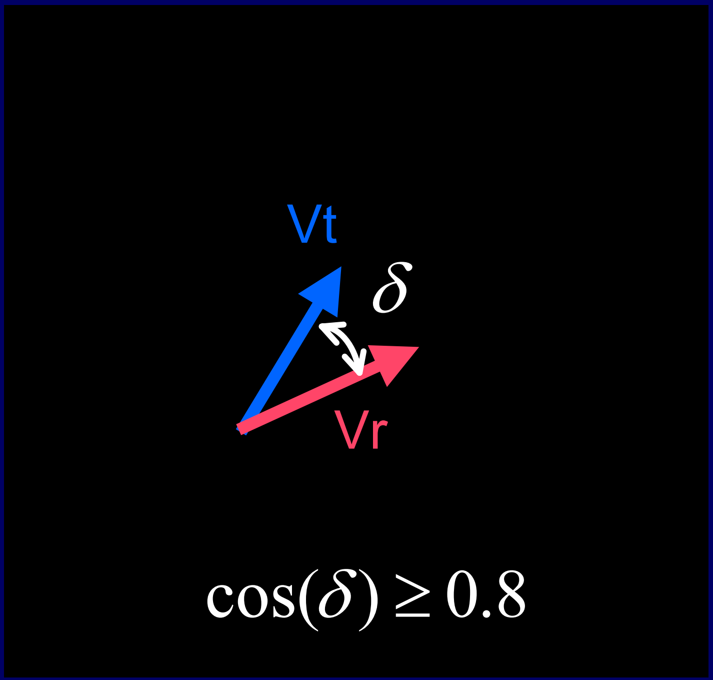
$$TraceADC \leq 0.003\ (mm^2/s)$$



Discrete Approach

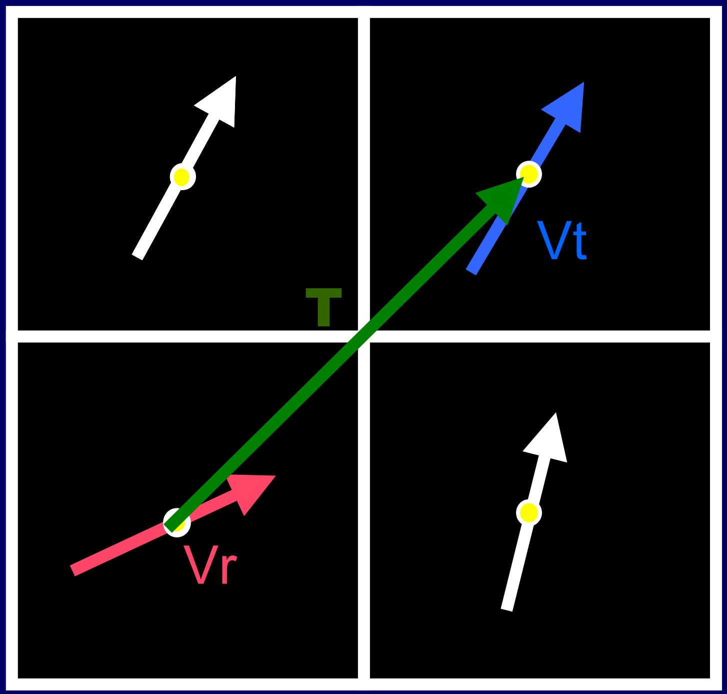
- Multi-voxel connectivity

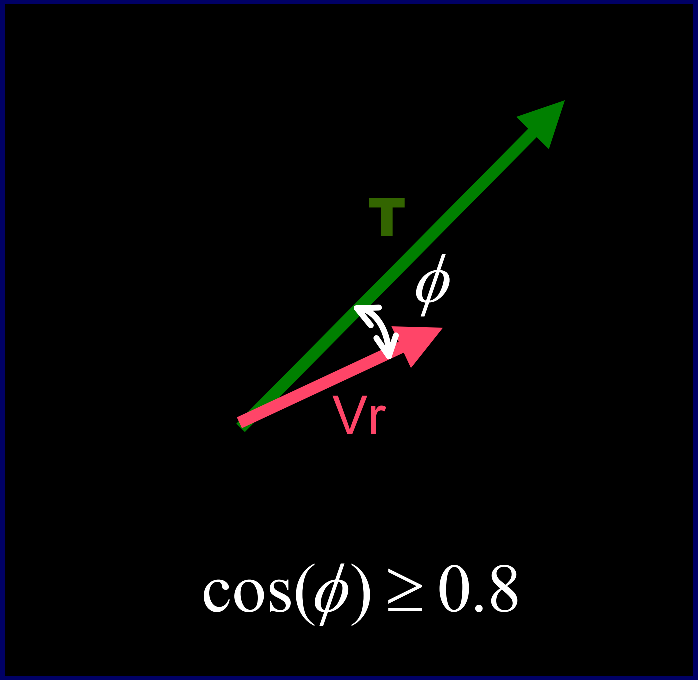




Discrete Approach

- Tract connectivity



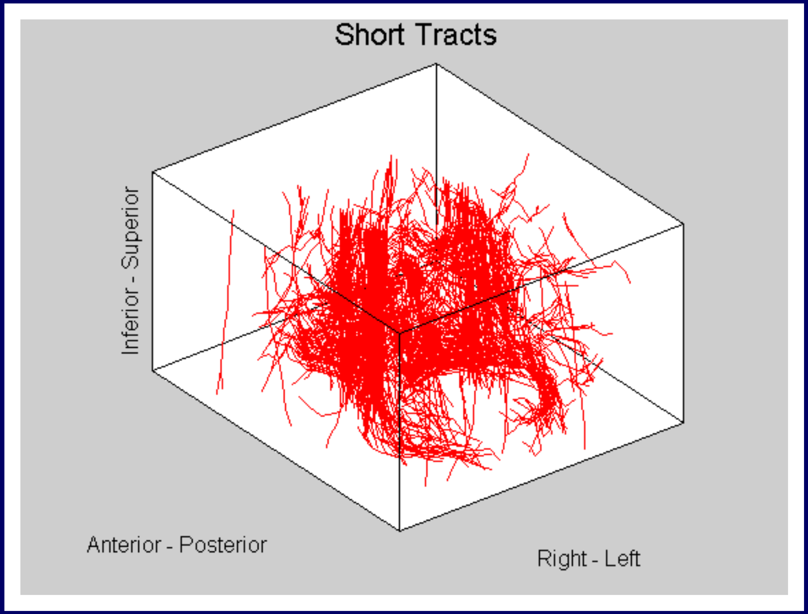


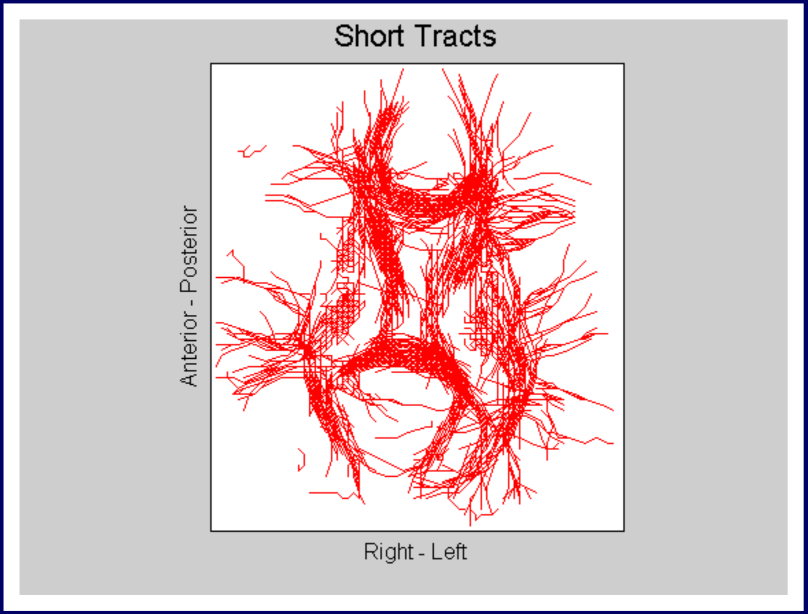
Tracing Criteria

- TraceADC $\leq$ 0.003 (mm^2/sec)
- Tan(θ) $\geq$ 1.2 ~ FA $\geq$ 0.1
- Cos(δ) $\geq$ 0.8
- Cos(Φ) $\geq$ 0.8

Discrete Approach

- EZ-Tracing tractography





Continuous Approach

- Fiber trajectory = 3D space curve r(s)
  - Tangent vector

$$\frac{d\mathrm{r}(s)}{ds} = t(s)$$

  - Key idea :

$$\frac{d\mathrm{r}(s)}{ds} = t(s) = \varepsilon_1(s)$$

Continuous Approach

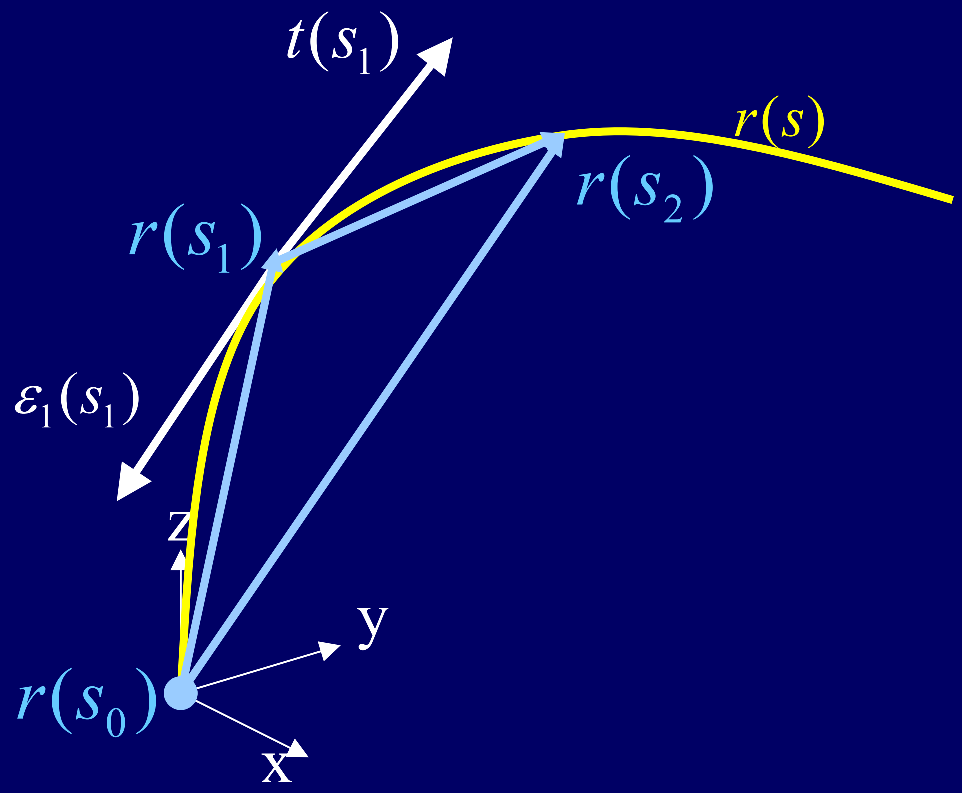
- Choose a starting point, $r(s_0)$
- Taylor expansion of $r(s)$ about $r(s_0)$

$$r(s_1) = r(s_0) + r'(s_0)(s_1 - s_0) + \ldots$$
$$\approx r(s_0) + \varepsilon_1(r(s_0)) \cdot \alpha \qquad \alpha << 1$$

- Choose $r(s_1)$ as the starting point, again and again…

Continuous Approach

- Fiber Tractography using DT-MRI data



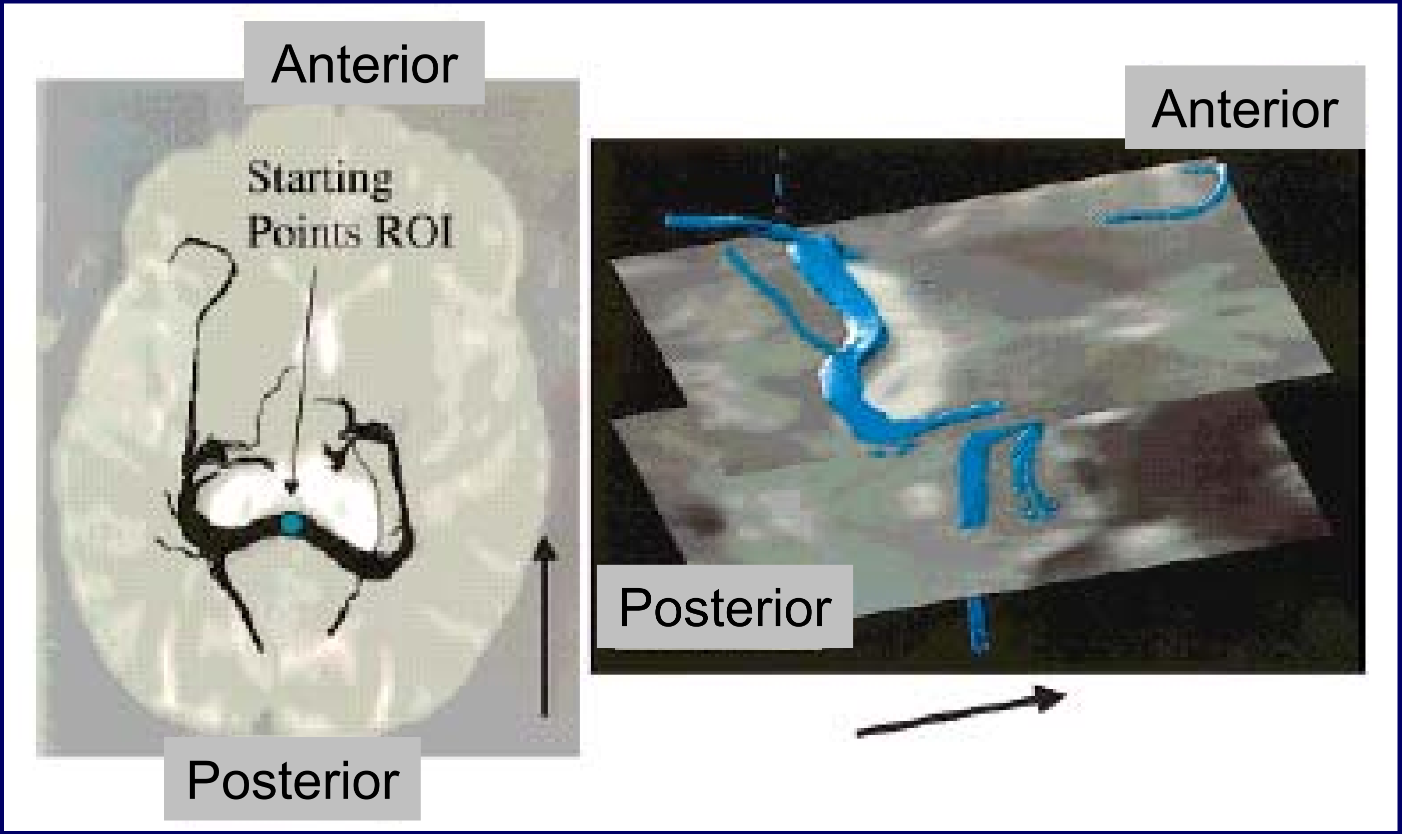
Tracing Criteria

- FA > 0.1
- Radius of curvature > 2 voxels

$$\kappa(s) = \frac{\left|d\mathtt{t}(s)\right|}{ds} = \frac{\left|d\left(\frac{r'(s)}{\left|r'(s)\right|}\right)\right|}{ds} < \frac{1}{2}$$

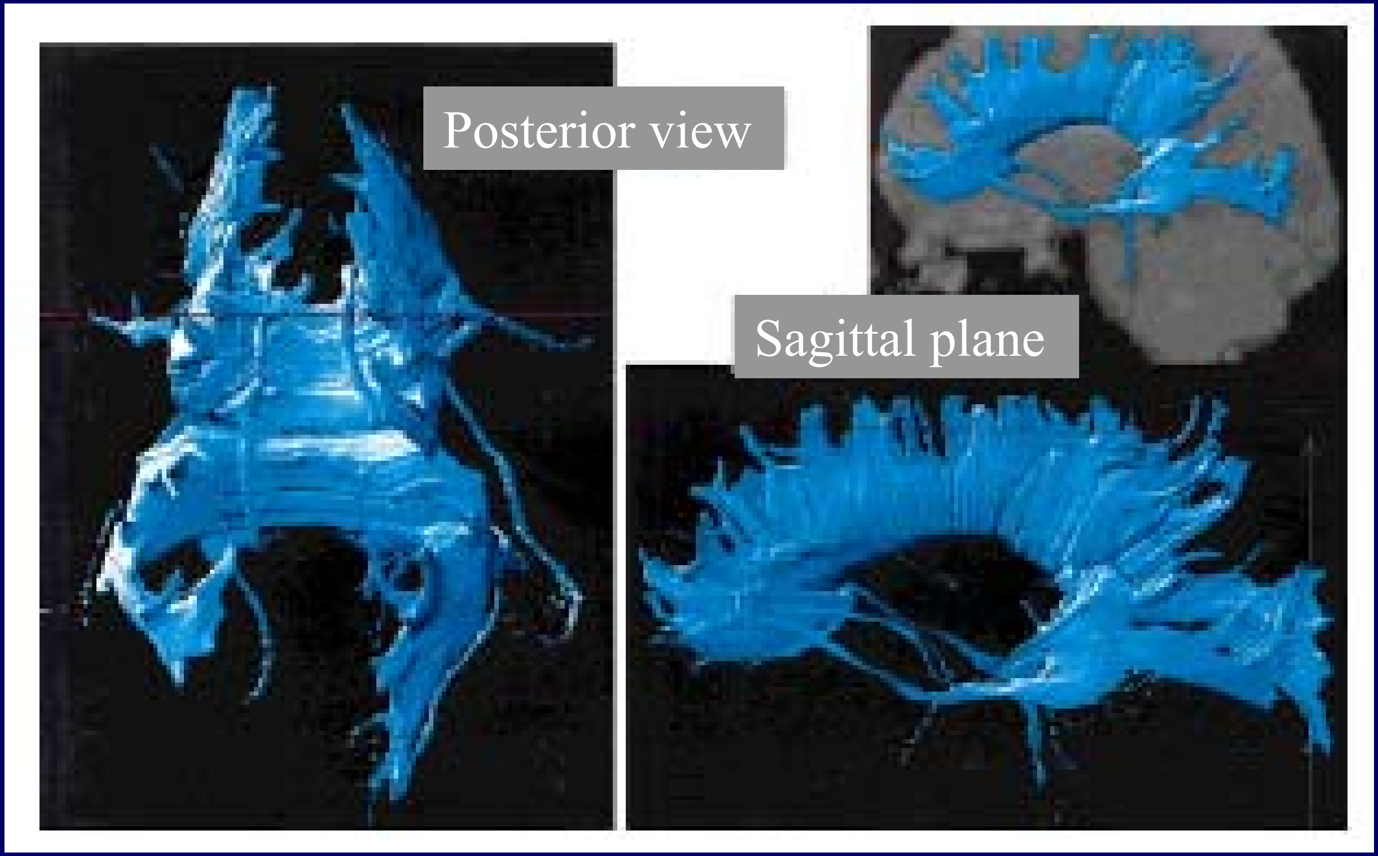
Continuous Approach

- Start from initial points ROI



Continuous Approach

- Fiber Tractography using DT-MRI data



Tractography

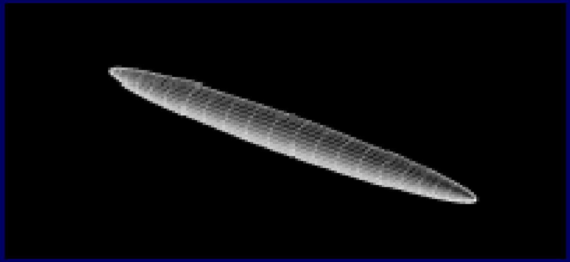
- EZ-Tracing : start from every voxel
  - Solve fiber-crossing
- F.T. : start from single voxel
  - Failure in crossing region
- Both assume single fiber in each voxel
  - Voxel contains multi-directional fibers

Limitations & Solutions

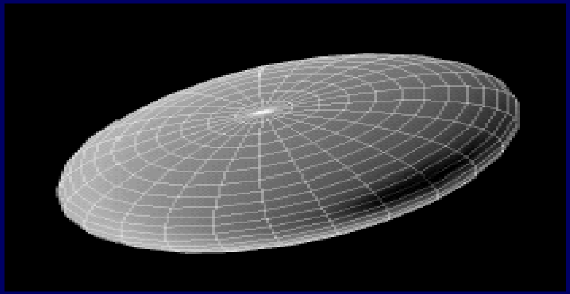
- Deviation from cigar-shape anisotropy
  - Tensorline method
- Fiber crossing
  - Start from every point
- Validation
  - Chemical tracer techniques

Deviation from cigar-shape anisotropy

- Cigar-shape $\lambda_1 >> \lambda_2, \lambda_3$



- Pancake-shape $\lambda_1 \approx \lambda_2 > \lambda_3$

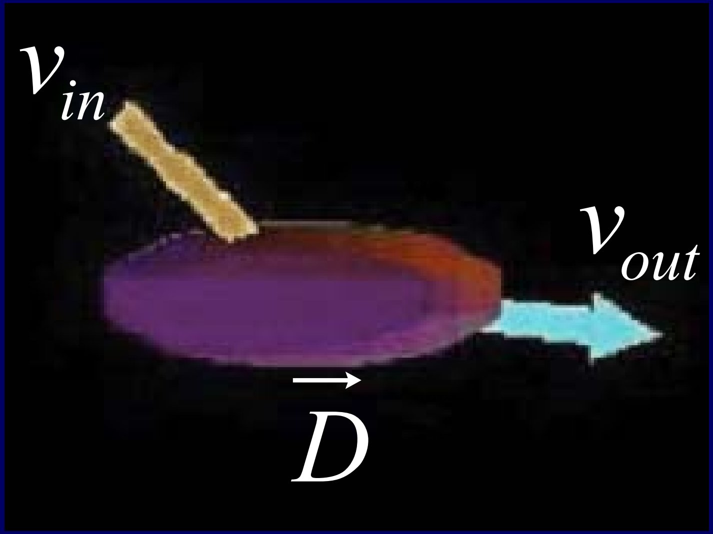


Solution

- Tensorline Technique, ISMRM 2000

$$v_{out} = \overrightarrow{D} \cdot v_{in}$$

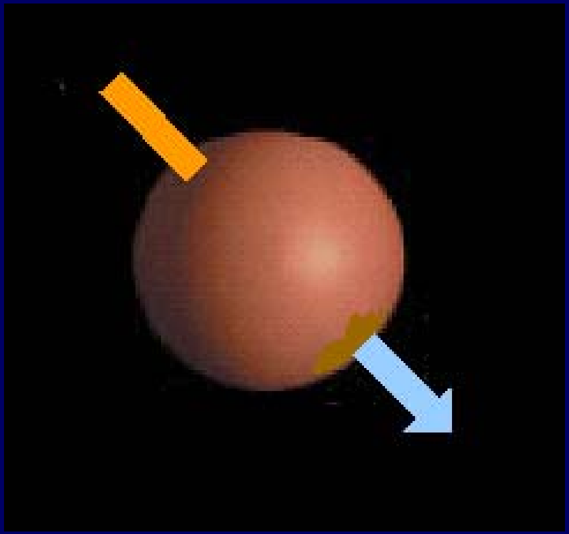
$$\begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}_{out} = \begin{pmatrix} D_{xx} & D_{xy} & D_{xz} \\ D_{xy} & D_{yy} & D_{yz} \\ D_{xz} & D_{yz} & D_{zz} \end{pmatrix} \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}_{in}$$

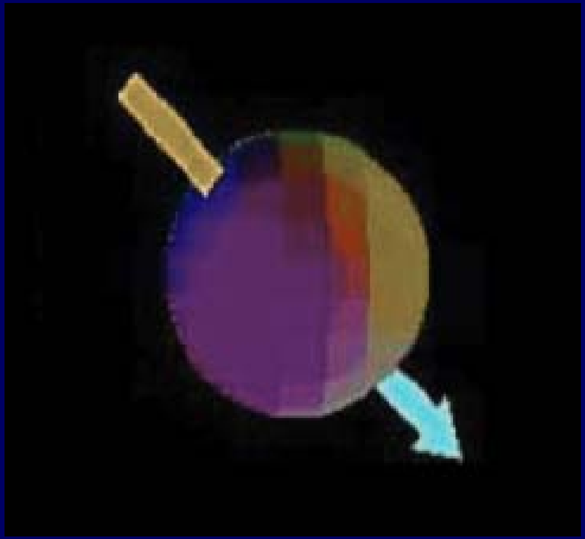


$$v_t = (1-\alpha)v_{in} + \alpha\, v_{out} \qquad \alpha = 0.1 \sim 0.6$$

Tensorline

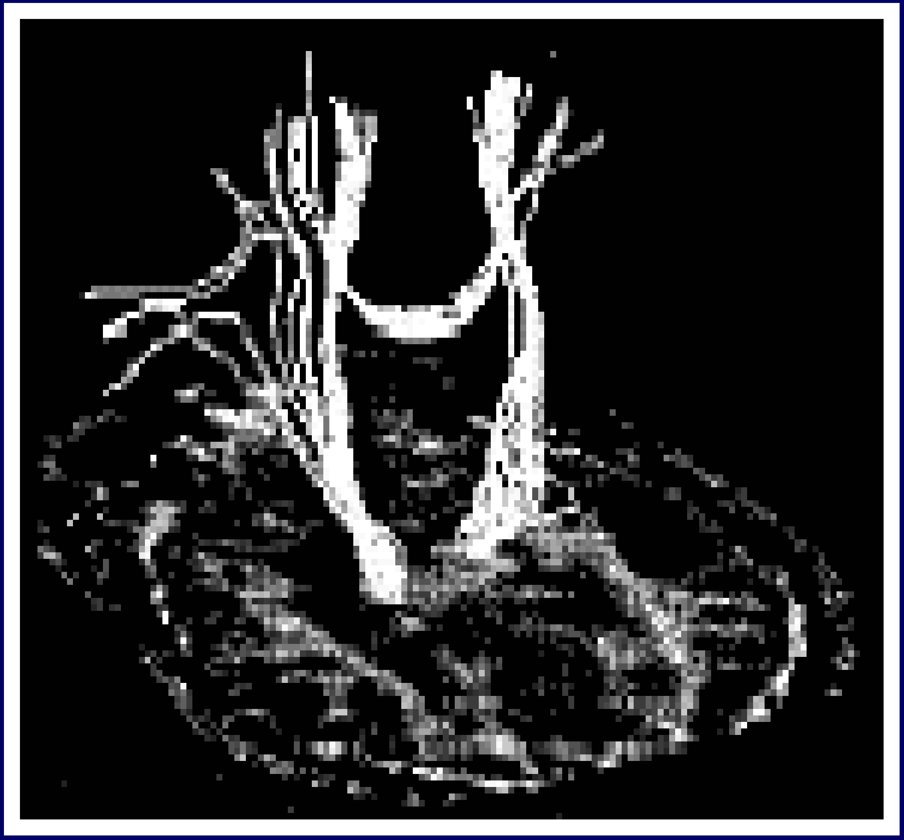
- When $\vec{D}$ is spherical or v_{in} is parallel to the flat plane of the pancake-shaped elliposid, $v_{out} = v_{in}$



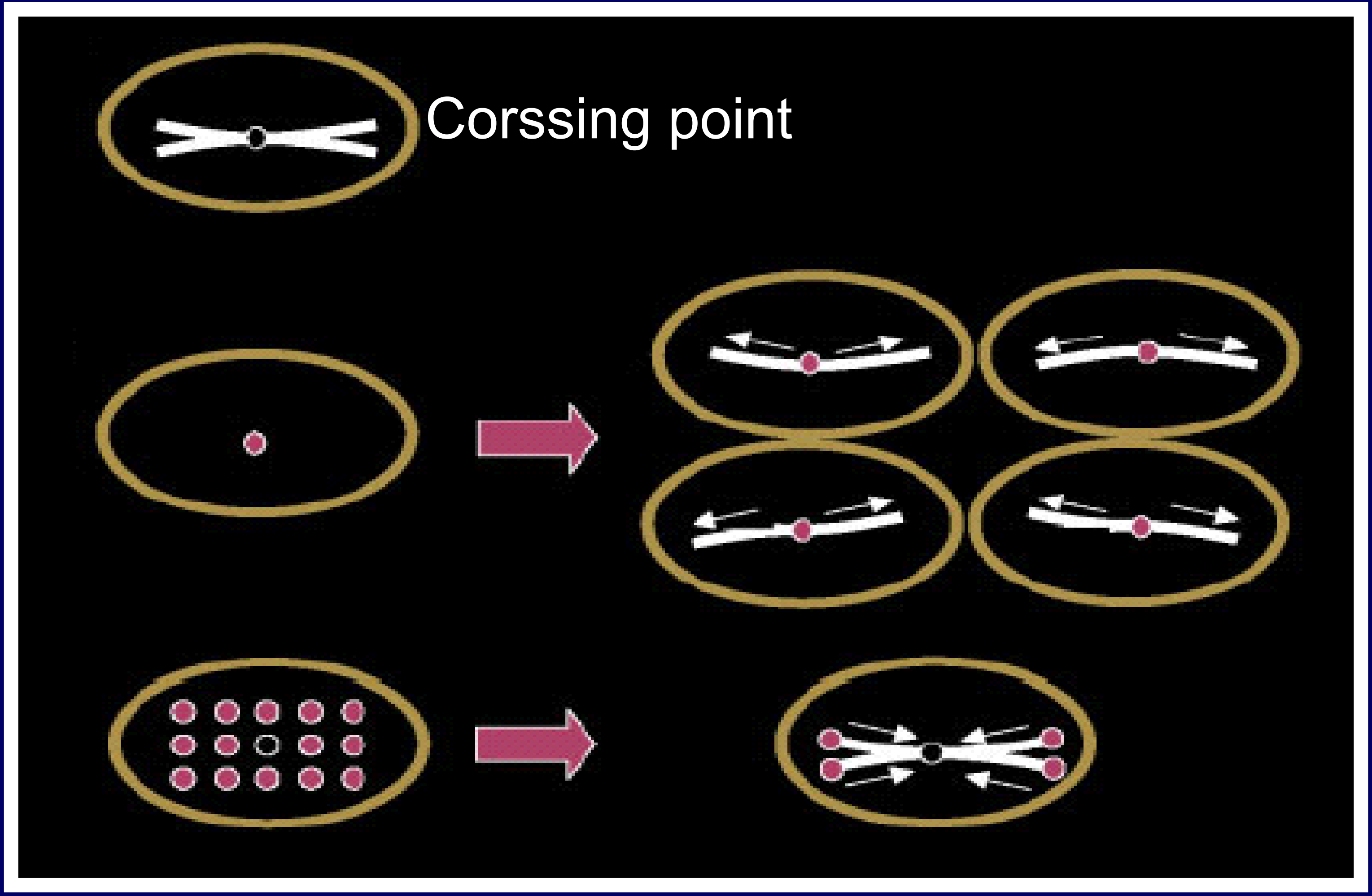


Tensorline

- Tensorline, ISMRM 2000



Fiber crossing

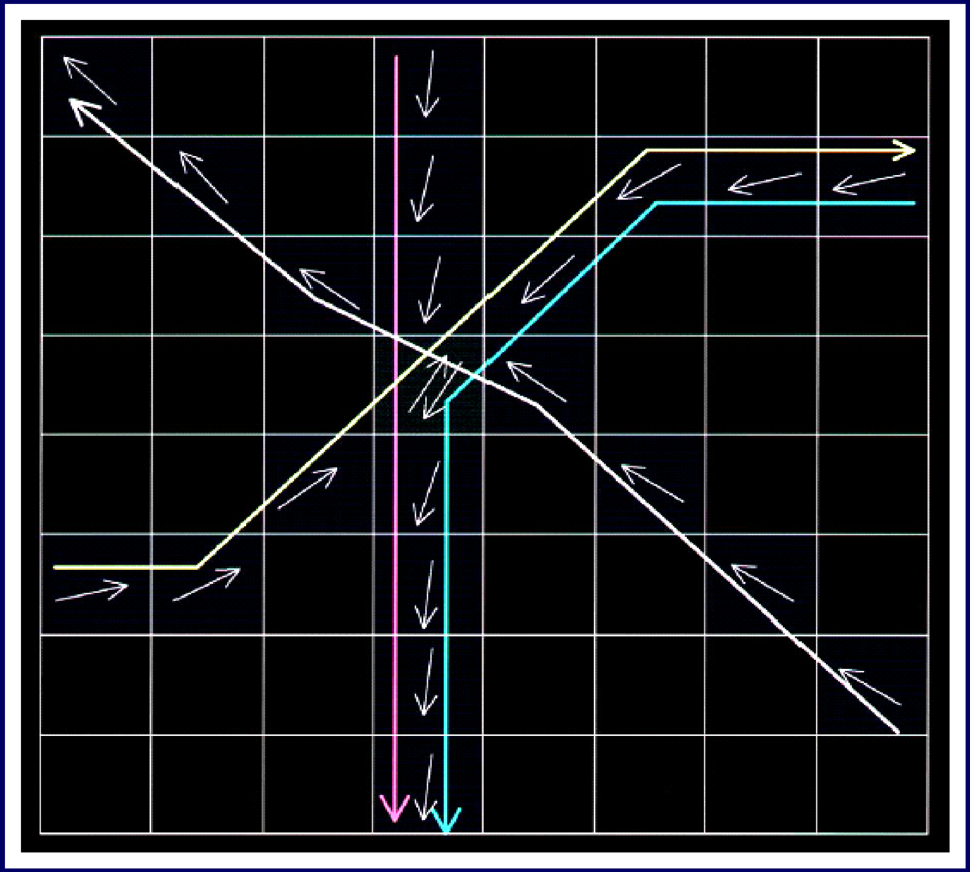


Solutions

- Start from multiple voxels

Solutions

- EZ-Tracing : start from every voxel



Validation

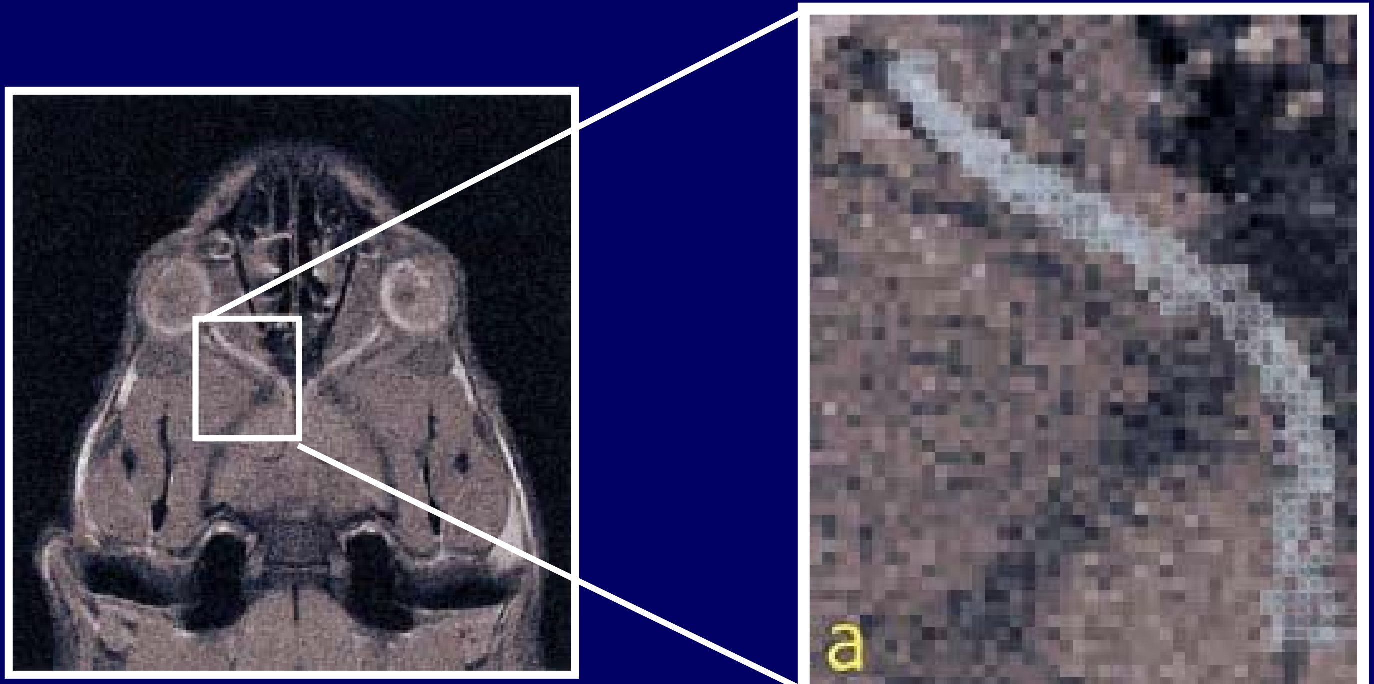
- Direct validation
  - Sample registration, Dissection, Freezing, Dehydration, Fixation, Microtoming, Thawing, etc.
  - Each will alter the microanatomy
- Solution
  - Chemical tracer technique

Solution

- Chemical Tracer Technique
  - Manganese ion (Mn^{2+})
  - T1-shortening agent
  - Uptaken actively by the axon
  - Toxic material

Chemical Tracer

- Mn^{2+}-enhanced MRI (T1WI) of mouse



Chemical Tracer

- Validation via chemical tracer technique

Conclusion

- Delineate the core of large white matter tracts.
- At present, it is the unique non-invasive technique to provide tract information.
- A powerful technique to investigate white matter anatomy and disease.